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July 1981

"BAYESIAN" RELIABILITY TESTS
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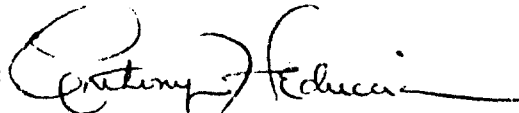
Anthony Coppola

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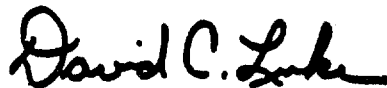
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This report provides an understandable, easy to use and believable approach to "Bayesian" Reliability Testing in order to encourage its practical application. It discusses advantages of the Bayesian approach, guidelines to selecting test risks, procedures for analyzing prior data and methods for structuring test plans. Sample requirements for invoking Bayesian test plans are included.

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1. INTRODUCTION.

"Bayesian" reliability tests have been a much discussed but seldom applied engineering technique. Part of the problem is a lack of understanding of the procedure, compounded by the promulgation of controversial approaches. However, even an approach well understood and accepted by the user may fall by the wayside because of mathematical complexity. It is the purpose of this report to provide an understandable, easy to use, and believable approach to "Bayesian" reliability testing in order to encourage its practical application.

"Bayesian" methods for reliability demonstration are procedures for incorporating existing data into a reliability demonstration plan. The term stems from Bayes' theorem of conditional probability and, strictly speaking, not all reliability test plans using prior data are correctly called Bayesian Reliability Tests. Hence, the quotation marks in the title of the report. For editorial simplicity, we will discontinue the use of the quotation marks and ask the reader to accept the term as referring to all test plans based on the use of prior data.

The interest in Bayesian test plans arises from its potential to permit shorter reliability tests at no loss of confidence. However, this advantage holds only if the prior is favorable, i.e. existing data on the type of equipment to be tested indicates a mean time between failure (MTBF) higher than required. It is quite possible for a Bayesian approach to result in a longer test than classical test methods if the prior is unfavorable. This point must be considered if the intent is strictly to reduce test time.

A less obvious advantage to Bayesian tests is the enrichment of the choice of statistical risks. In classical test plans, the statistical risks are the consumer's risk (the probability of accepting an equipment if its actual MTBF equals a specific value defined as undesired) and the producer's risk (the probability of rejecting an equipment if the actual MTBF equals a specific value defined as desired). Using Bayesian test plans, several other definitions of risk are possible, resulting in a selection of those risks most meaningful to the circumstances.

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In the following sections we will compare the Bayesian approach to classical reliability demonstration, examine some of the Bayesian tools available, and develop a suggested approach.

2. BAYESIAN VS. CLASSICAL PLANS

Figure 1 shows the flow for developing a classical test plan. The responsible engineer starts by determining what values of producers and consumers risks are acceptable to him. He has no choice of the type of risks involved, only of their values. This of course implies defined values for a desired MTBF (the upper test MTBF, θ_0, as defined in MIL-STD-781C, "Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution") and for an undesirable MTBF (the lower test MTBF, θ_1, as defined in MIL-STD-781C). He then selects a test plan from MIL-STD-781 which satisfies these risks, or, if he wants to bother, creates one with the aid of various statistical tools such as the Wald sequential test formulas, or the Poisson and chi-square tables. He must also determine the test environment and monitoring procedures, a non-trivial task fortunately outside the scope of the report. Completing these tasks, he has the required test plan. As Figure 1 shows, any prior data which may exist is completely ignored by the process.

Figure 2 shows the process of creating a Bayesian test. The responsible engineer must now select the type of risks he desires as well as their values. To use the prior data he must analyze it and extract the statistical parameters of interest. These parameters with his selection of risk types and values are used to structure the test plan. Adding the test environment and monitoring procedures, as before, completes the job.

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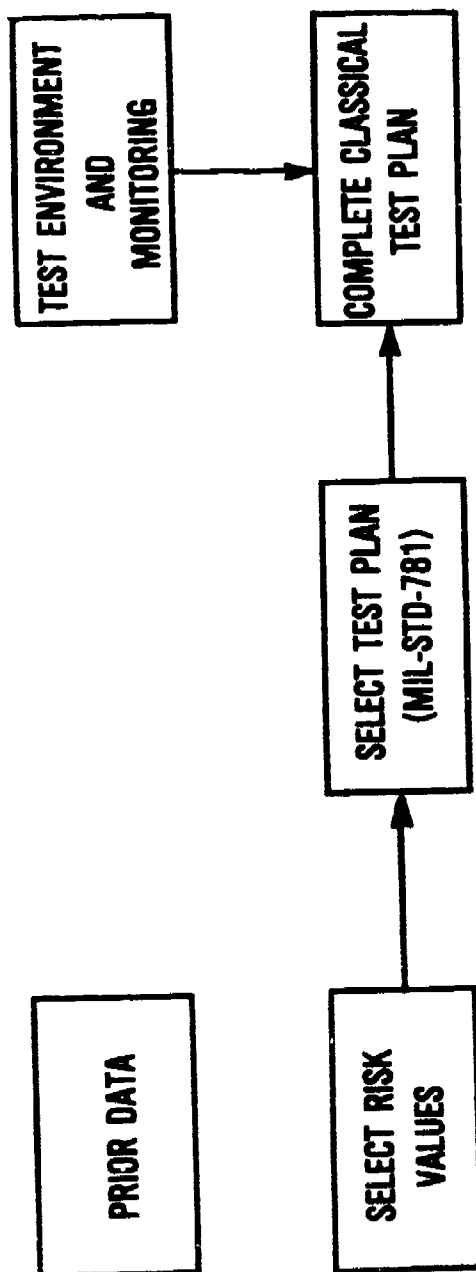
DEVELOPING A CLASSICAL TEST PLAN

FIGURE 1
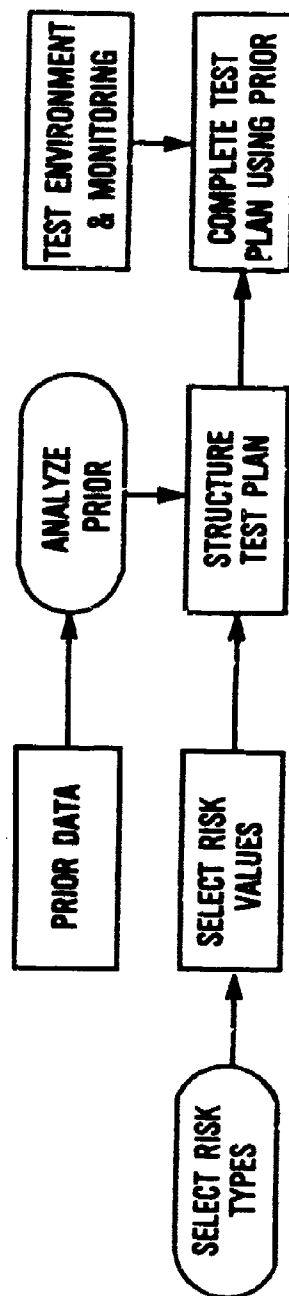
FIGURE 2

DEVELOPING A BAYESIAN TEST PLAN

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At this point the reader may wonder if it's worth the effort. Our intent is of course to reduce the effort, but even a difficult effort might be worthwhile. Advantages of the Bayesian approach are:

(a) A favorable prior will permit a shorter test with attendant savings in dollars and time. For example, a commercial computer may be purchased for a military command, control and communications system. If it has a good history based on solid and pertinent data, is it worth the time and cost of a classical test when a Bayesian test can provide the results quicker? The only other option is a complete waiver of testing, which is not recommended, since a particular unit may be a lemon even if the average unit is great.

(b) Used for production reliability control, each test performed can use all the previous test data as prior information. If a downward trend is indicated, the test plans will become more stringent. If the trend is towards improved reliability, the tests become easier. Hence, the test severity is self-regulating, providing the proper test length and accept criteria for the data available at all times. The alternatives are low risk classical tests with greater cost penalties or high risk classical tests, which cost less, but could be too liberal in a declining reliability trend.

(c) Using classical tests for production control compounds the producers risks. Assuming his equipment were of a constant high quality such that it would pass a classical test with 10% risk (90% probability of acceptance), and four tests were scheduled during production, his probability of passing all four tests would be $(.90)^4 - .6481$, which is getting uncomfortable. With a Bayesian approach, continued high quality would reduce the risk of rejecting good equipment as it would successively reduce the stringency of the test.

(d) The Bayesian approach allows the use of better risk definitions. For example, suppose a classical test provides a 5% probability that a poor MTBF would be accepted. Suppose all equipment produced had a poor MTBF. Only 5% of

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the units tested would be accepted, but 100% of the accepted units would be poor. With Bayesian test plans, instead of the classical consumers risk, a posterior risk can be used. This is defined as the percent of accepted equipment which is poor, which is obviously a better choice in an environment of generally poor quality.

The next sections will look at the various steps in the Bayesian test generation process, and will try to provide a simple approach to each.

3. SELECTING TEST RISKS

Classical reliability test plans are based on two statistical risks.

The producers risk (α) defined as the probability of rejecting a system whose MTBF is equal to a "desired" value θ_0. The "desired" value has been called the "specified" value and is referred to in MIL-STD-781C as the upper test limit.

Mathematically: $\alpha = P(R|\theta_0) = 1 - P(A|\theta_0)$ (3.1)

where $P(R|\theta_0)$ is the probability of rejection given that θ_0 is the true MTBF of the test sample, and

$P(A|\theta_0)$ is the probability of acceptance.

The consumers risk (β) is the probability of accepting a system whose MTBF is equal to an "undesired" value θ_1. The "undesired" value has been called the "minimum acceptable" value and is currently labeled in MIL-STD-781C as the lower test limit.

or: $\beta = P(A|\theta_1)$ (3.2)

where $P(A|\theta_1)$ is the probability of accepting a sample with a true MTBF of θ_1.

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The need for θ_0 and θ_1 stems from the fact that reliability tests are basically designed to discriminate between two alternative hypotheses (equipment is good vs equipment is bad), which must be numerically defined. Figure 3 is an operating characteristic curve which plots the probability of acceptance against the true MTBF of the test sample for a typical reliability test. The producer's risk (the probability of rejecting an equipment with true MTBF of θ_0) is one minus the distance from the horizontal axis is to the point on the curve directly above θ_0. The consumer risk is simply the distance from the horizontal axis to the curve above θ_1. Note the shaded areas labelled $\bar{\alpha}$ and β. These represent respectively the probability of rejecting an equipment with θ_0 or higher MTBF and the probability of accepting an equipment with θ_1 or lower MTBF. These risks are called the average risks and designated by $\bar{\alpha}$ and β. They probably more accurately reflect the desires of the test engineer, but cannot be computed without the analysis of prior data (or an assumption of the nature of the population of the equipment to be tested).

The average producers risk ($\bar{\alpha}$) can be defined as:

$$\bar{\alpha} = \frac{\text{number of good systems tested that are rejected}}{\text{total number of good systems tested}}$$

Similarly:

$$\beta = \frac{\text{number of bad systems that are accepted}}{\text{total number of bad systems tested}}$$

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FIGURE 3

OPERATING CHARACTERISTIC CURVE

PROBABILITY OF ACCEPTANCE

1.0

$\bar{\alpha}$

α

β

$\bar{\beta}$

θ_0 ("GOOD")

θ_1 ("BAD")

TRUE MTBF
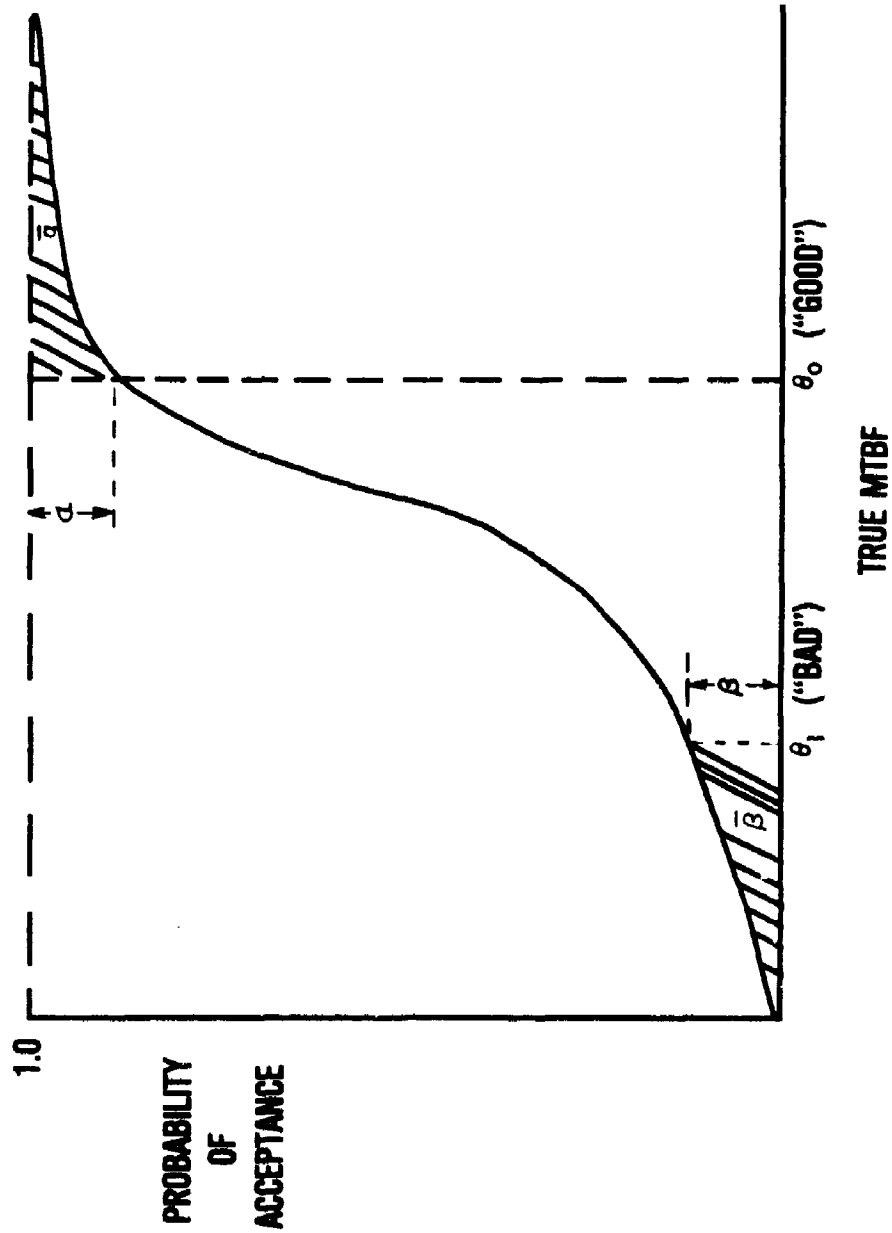
Obviously, to compute $\bar{\alpha}$ and $\bar{\beta}$ requires a knowledge of the total number of good and bad systems, and hence cannot be done with classical procedures. Using Bayesian procedures, if we can define a probability density function $g(\theta)$ for the equipment MTBF we can define the risks mathematically as follows:

Where $P(R|\theta)$ = probability of rejecting an equipment with MTBF of θ

$$\bar{\alpha} = \frac{\int_{\theta_0}^{\infty} P(R|\theta)g\theta\, d\theta}{\int_{\theta_0}^{\infty} g(\theta)\, d\theta} \qquad (3.3)$$

or

$$\bar{\alpha} = \int_{\theta_0}^{\infty} P(R|\theta)\, g(\theta|\theta \geq \theta_0)\, d\theta \qquad (3.4)$$

similarly:

$$\bar{\beta} = \int_{0}^{\theta_1} P(A|\theta)\, g(\theta|\theta \leq \theta_1)\, d\theta \qquad (3.5)$$

where $P(A|\theta)$ is the probability of accepting a unit with MTBF of θ.

Another set of risks possible with Bayesian approaches is the posterior risks (α^*, β^*). In the frequency sense:

α^* = $\dfrac{\text{number of good systems among the rejected system}}{\text{total number of rejected systems}}$

β^* = $\dfrac{\text{number of bad systems among the accepted systems}}{\text{total number of accepted systems}}$

mathematically:

$$\alpha^* = \frac{\int_{\theta_0}^{\infty} P(R|\theta) \, g(\theta) \, d\theta}{\int_0^{\infty} P(R|\theta) \, g(\theta) \, d\theta} \qquad (3.6)$$

$$\beta^* = \frac{\int_0^{\theta_1} P(A|\theta) \, g(\theta) \, d\theta}{\int_0^{\infty} P(A|\theta) \, g(\theta) \, d\theta} \qquad (3.7)$$

Another risk of possible interest is the probability of rejection, $P(R)$. This is a single number defined as:

$$P(R) = \frac{\text{Total number of systems rejected}}{\text{Total number of systems tested}}$$

or

$$P(R) = \int_0^{\infty} P(R|\theta) \, g(\theta) \, d\theta \qquad (3.8)$$

Note this risk is independent of θ_0.

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Other risks are possible such as the alternate posterior consumers risk (β^{**}) defined as the probability of an accepted product being below θ_0.

$$\beta^{**} = P(\theta \le \theta_0 | A) = \int_0^{\theta_0} P(\theta | A) \, d\theta \qquad (3.9)$$

or

$$\beta^{**} = \frac{\int_0^{\theta_0} P(A|\theta) \, g(\theta) \, d\theta}{\int_0^{\infty} P(A|\theta) \, g(\theta) \, d\theta} \qquad (3.10)$$

With this wealth of risk options, the analyst may well wonder which he should use. The choice, ideally, is two risk types which best fit the real interests of the producer and consumer. These interests will vary according to the way requirements are defined, the relation between the number of units produced and the number procured by the consumer, and the willingness of the producer and consumer to accept a prior distribution.

For this discussion we will assume that requirements are defined in terms of θ_0 and θ_1. Philosophically, θ_0 should represent the highest value of MTBF believed achievable under the state-of-the-art and the required reliability effort and θ_1 should be the lowest value the consumer would be willing to accept from both mission success and supportability viewpoints. Hence, equipment with an MTBF of θ_0 (or better) should have a high probability of acceptance and equipment with θ_1 (or less) should have a high probability of rejection.

These requirements suggest the use of average risks. However, the quantities produced and bought can make other risks more desirable.

Consider first the case where the producer produces a large number of systems and the consumer buys a large number of systems. A recent practical example is the AN/ARC-164 program, where over 10,000 airborne UHF radios were produced for the Air Force. Tests based on prior distribution could be used in

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such programs for production reliability control, with the original prior provided by data from the reliability qualification tests, and the subsequent priors derived by combining the original prior with the results of each production test performed.

In this case, the desirable risks are β^*, $P(R)$. The consumer does not have to limit himself to β which tells him only the probability that a unit of actual MTBF θ_1 will fail the test. $\bar{\beta}$ tells him the probability of units with θ_1 or less being accepted which is more meaningful. However, taking an extreme case, suppose all units produced were below θ_1. Then no matter how low β or $\bar{\beta}$ were, all accepted items would be bad. Hence, β^* which tells him the percent of accepted units which will be bad, is the most meaningful figure. For similar reasons, the consumer, who does not want to reject good units, would prefer α^* to α or $\bar{\alpha}$ as the producer's risk. However, the producer desires to have all the units produced be acceptable and hence, $P(R)$, representing the percent of units rejected is his most pertinent risk criterion.

Now, suppose the producer produces a large number of systems and the consumer buys a small number. An example of this could be the purchase of a commercial computer for use in an Air Force Command and Control system. Here a test based on prior distribution could be used for acceptance of the computer with the prior derived from the history on the producer's products sold to previous customers. Another example is the purchase of commercial test equipment for Air Force use.

For this case, the desirable risks are: $\bar{\alpha}$, $\bar{\beta}$. Since the consumer is procuring a small quantity, perhaps only one, he is concerned with the risk that the lot will be poor equipment. The conventional risk β would do, but $\bar{\beta}$ more accurately describes his concern. The producer could be interested in $P(R)$ interpreted as the probability of a random system being rejected. However, since he has other customers to whom he can sell the rejected equipment and presumably does not want the stigma of selling an equipment that did not meet the customer's requirement, $\bar{\alpha}$ would better define his risk.

Another possibility is that a specific producer produces a small number of systems and the consumer buys a large number (from different producers). At

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present, this case is not of great interest. It arises when there is a need for vast quantities of equipment beyond the capabilities of any one supplier, such as seen during World War II, when many suppliers produced the same equipment for the war effort. The test would be used to accept the products of the individual consumers with the prior established from the first producer and updated as information became available from other producers. In this case, the consumer is still acquiring a large number of units and hence, would prefer β^* to describe his risk. The producer, however, has a small number to sell and would find $\bar{\alpha}$ his most meaningful risk.

Finally, suppose the producer produces a small number of systems and the consumer buys a small number. This is a frequent case in Air Force applications. Ground systems can be procured in quantities as small as one (e.g., the AN/FPS-85 radar), and Command and Control systems are typically composed of a few (although often large) components. In space applications even an extensive satellite system is composed of a statistically small population, and even airplane systems can be provided in small quantities (e.g., the Advanced Airborne Command Post). In these cases, tests based on prior distribution could be used for acceptance of the equipment, if a suitable prior exists. However, a small production, especially of large components, implies a small pre-production test sample and the prior may be hard to come by. An exception would be the purchase of a small lot of equipment to add to an existing inventory (e.g., the addition of a new site to an existing radar system) where sufficient previous test data should be available. If a prior exists and is acceptable to both, $\bar{\alpha}$ $\bar{\beta}$ would apply since both producer and consumer are concerned with a small lot.

The preceding discussion assumes a prior exists and is acceptable to both producer and consumer. If no prior exists, there is no choice but to revert back to the classical risks (α β).

Other possibilities include the acceptance of different priors by producer's & consumers and the acceptance of a prior by one but not the other. It is possible to structure test plans where producer's and consumer's risks are based on different priors or to create plans where one risk is based on a prior and the other is a classical risk. These hybrid tests should be acceptable to

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both producer and consumer, since each one can have the risks he wants. For example, when the consumer has a prior and the producer does not, the producer must fall back on the conventional risk α. The consumer can specify a test using, say, β^* as his risk. Since the contractor has the same risk as the conventional test would provide, his lack of agreement on the prior should not affect his willingness to accept the test. Where there is no agreement on the prior, a test may be specified using risks based on the different priors. This should be a rare situation as test plans are specified by the consumer before contract award, but it could conceivably arise in negotiations for a contract. Whether or not the test provides any benefits in reduced test time depends on the prior.

While such hybrid approaches are possible, they are by no means simple. Since the object of this report is to simplify the use of Bayesian test techniques, our final recommendations will not include hybrid test approaches.

The following matrix may be used to select the most desirable test risks as a function of equipment quantities.

	Large Production		Small Production	
	Large Buy	Small Buy	Large Buy	Small Buy
Producers Risks	$P(R)$	$\tilde{\alpha}$	$\tilde{\alpha}$	$\bar{\alpha}$
Consumers Risks	β^*	$\tilde{\beta}$	β^*	$\bar{\beta}$

Finally, note that while the risks shown in the matrix are the most desirable, in a practical situation almost any combination of risks would still be meaningful. The simplest approach would be to use the average risks $\tilde{\alpha}\tilde{\beta}$

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which are the closest to the classical risks. In fact, the classical risks are really a surrogate for the average risks, used because the classical approach permits no other.

4. ESTIMATING THE PRIOR

At this time, two points must be established. First, the statistical assumptions on which this procedure is based and, secondly, the philosophy of using priors.

There are two statistical assumptions underlying the development in this report. These are:

(a) The prior distribution is an inverted gamma.

(b) The times between failures follow an exponential distribution (i.e., a constant failure rate exists).

The inverted gamma prior density is assumed because it is a flexible distribution capable of representing many practical situations, it has properties facilitating mathematical manipulation, and, most importantly, it seems to fit the real world. (Reference RADC-TR-76-294, Vol III "Reliability Acceptance Sampling Plans Based Upon Prior Distribution, Implications and Determination of the Prior Distribution").

An inverted gamma distribution is illustrated in Figure 4. Its formula is:

$$g(\theta) = \frac{\gamma^\lambda \theta^{-(\lambda + 1)} e^{-\gamma/\theta}}{\Gamma(\lambda)} \qquad (4.1)$$

where θ, γ, $\lambda > 0$

properties are:

$$\text{mean} = \frac{\gamma}{\lambda - 1}$$

$$\text{variance} = \frac{\gamma^2}{(\lambda-1)^2(\lambda-2)}$$

$$\text{mode} = \frac{\gamma}{\lambda + 1}$$

$$g(\theta) = \gamma^\lambda \theta^{-(\lambda+1)} \exp(-\gamma/\theta)/\Gamma(\lambda)$$

INVERTED GAMMA DENSITY FUNCTION

λ (SHAPE PARAMETER) = 3.5, γ (SCALE PARAMETER)=470

FIGURE 4

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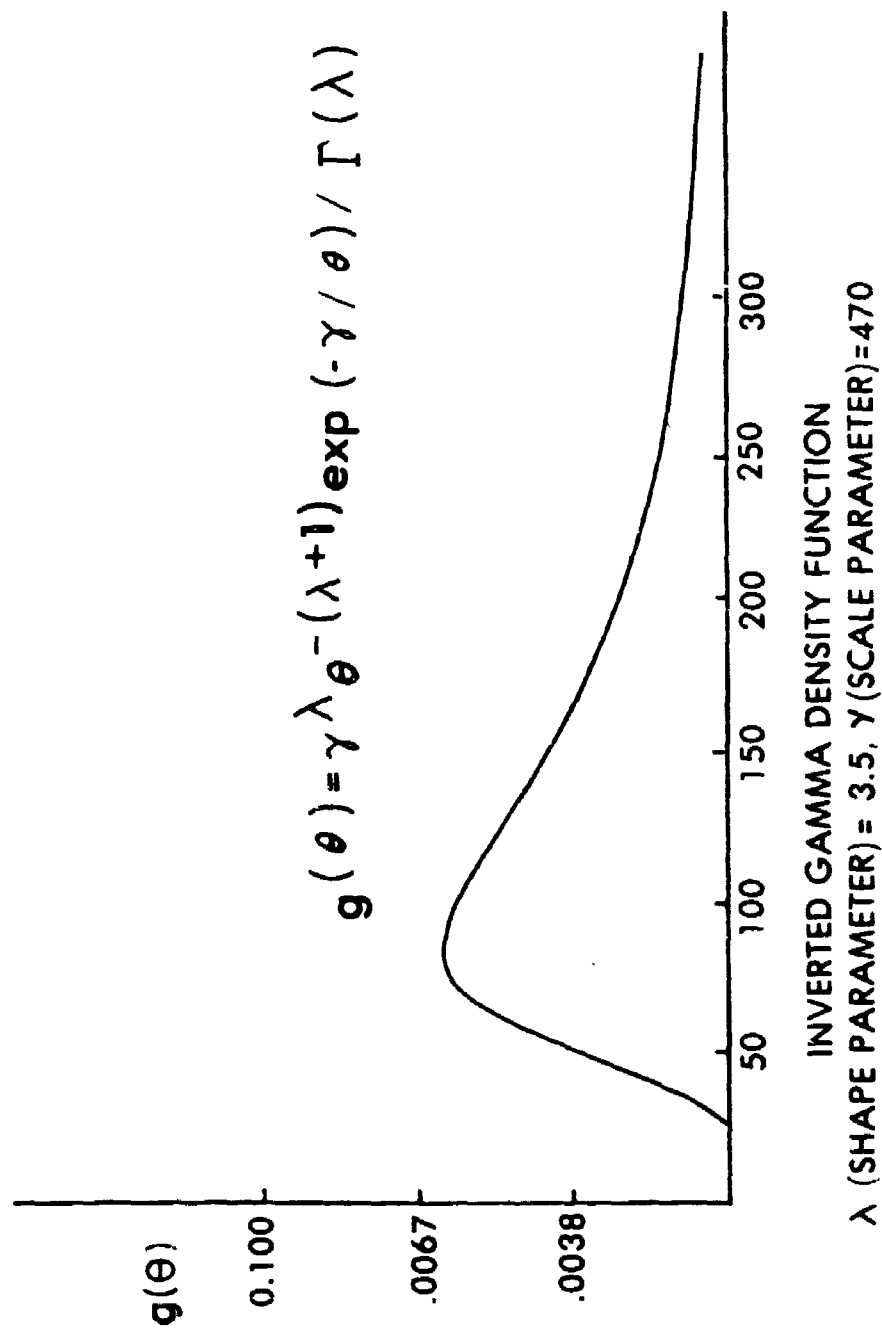
The constant failure rate is the standard assumption used in reliability demonstrations. It also has properties facilitating its mathematical handling. Of late, there have been some questions raised as to its portrayal of the real world. The alternate proposal is most often a decreasing failure rate with time, with the Weibull distribution replacing the exponential in describing time to failure. While these questions remain unresolved the exponential is still the standard for reliability demonstration and its use does represent a conservative approach (i.e., assuming a constant failure rate when a decreasing failure rate actually exists, would lead to somewhat pessimistic conclusions). Since equipment in actual usage in general appears to have a lower reliability than indicated by reliability tests, a conservative approach to the test has merit.

On the philosophy of using priors, let it be clearly understood that in this procedure only actual test data is considered a usable prior, and only then if it exists in sufficient quantity. We do not subscribe to "subjective" priors (i.e., what one believes the prior is, without data) or the use of reliability predictions as priors. (The prediction establishes a baseline which the equipment may or may not have achieved). Hence, only hard data may be used for the prior and a reversion to classical testing made when sufficient hard data is not available.

Considering the above, the task of estimating the prior consists of fitting available data to an inverted gamma distribution and solving for the parameters of the distribution.

The analyst's job is to determine the λ and γ of the formula that fits his data. This is simple to say, but much harder to realize.

RADC-TR-76-294, Vol III, referenced earlier examines this problem and various solutions. The report recommends the use of a maximum likelihood estimation procedure for determing λ and γ from the prior. This procedure requires the use of a computer program which is provided. A simpler technique, called

the Generalized Maximum Likelihood Method was developed later and published in RADC-TR-79-121, "Application of Bayesian Techniques to Reliability Demonstration, Estimation and Updating of the Prior Distribution."

The procedure is based on the function

$$W(\gamma) = \sum_{j \geq 1} \frac{\alpha_j}{\lambda(\gamma) + j - 1} - \sum_{i=1}^{n} \ln\left(1 + \frac{T_i}{\gamma}\right) \tag{4.2}$$

when

$$\lambda(\gamma) = \frac{\gamma \sum_{i=1}^{n} \frac{r_i}{T_i + \gamma}}{\sum_{i=1}^{n} \frac{T_i}{T_i + \gamma}} \tag{4.3}$$

and

$$\alpha i = \sum_{r_i \geq j} (1) \text{ (i.e. add one for every } R_i \geq j) \tag{4.4}$$

r_i = number of failures in the ith test sample

T_i = total test time of the ith sample

This is illustrated in Figure 5.

The analyst's job is to find the value of γ where W crosses the axis (W = 0). This is done by an iterative method, as follows:

Step 1 - a determination is made on whether or not the data is sufficient to establish a prior. The conditions for a solution are:

$$\bar{r} < \frac{1}{n} \sum_{i=1}^{n} \left(r_i - \frac{\bar{r}}{\bar{T}} T_i\right)^2 \tag{4.5}$$

where n is the number of test samples, $\bar{r}$ and $\bar{T}$ are the mean number of failures and mean test time for the sample, r_i and T_i are the number of failures and test time of the ith unit.

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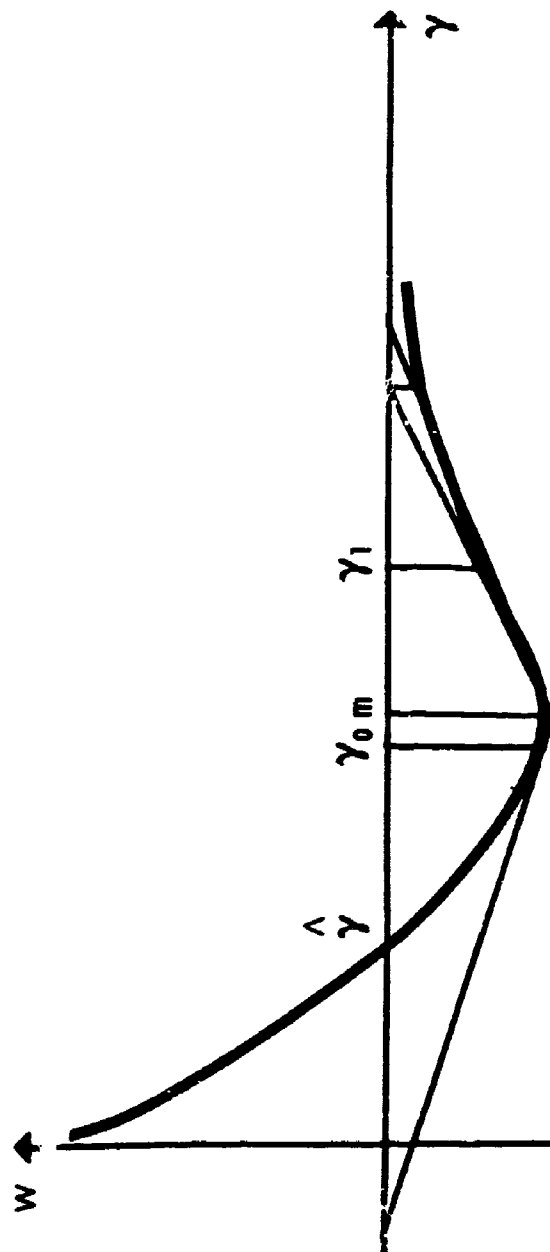
FUNCTION TO DETERMINE γ

FIGURE 5

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If this inequality does not hold, data is insufficient to establish a prior.

Step 2 - An arbitrary value for γ is selected and the equation for W solvcd. If W is a negative number, a smaller value of γ must be selected such as trying half the first value until a positive W is obtained. This is because the third and successive values of γ are extrapolated and if the first two are obtained where W is negative a final gamma of negative or infinite value can result (see tangent lines for γ_0 & γ_1 on Figure 5). If the value of W is positive with the first selection of γ then a higher value of γ is selected (try twice the first value) and W computed again.

Step 3 - From the first two values of γ a third value is extrapolated by

$$\gamma_3 = \gamma_2 - \frac{(\gamma_2 - \gamma_1) \, W \, (\gamma_2)}{W \, (\gamma_2) - W \, (\gamma_1)} \qquad (4.6)$$

W is computed for γ_3. γ_3 and γ_2 then become γ_2 and γ_1 respectively, for the next iteration.

Step 4. The process continues until the change in γ is negligible or W is satisfactorily close to zero.

Step 5 - λ is computed from equation 4.3.

This procedure can be done on a pocket computer but can be tedious. For those with access to a computer terminal the following FORTRAN program may be helpful.

With the parameters of the prior on hand, the analyst is ready for the next step, structuring the test plan.

FORTRAN PROGRAM FOR COMPUTING A PRIOR

```
 dimension r(1000),t(1000),a(1000)
****** input data in interactive mode ******
print, "enter number of units tested"
read, n
xn=n
print, "enter number of failures, test time for each unit; one unit at a time"
do 10 n1=1,n
read, r(nl),t(nl)
10 continue
****** test for suffici ncy of data ******
******RADC-TR-79-121,formula 3.3.1 ******
sumr=0;sumt=0
do 20 n2=1,n
sumr=sumr+r(n2)
20 continue
do 30 n3=1,n
sumt=sumt+t(n3)
30 continue
rbar=sumr/n;tbar=sumt/n
sc=0
do 40 n4=1,n
scl=(r(n4)-(rbar/tbar)*t(n4))**2
40 sc=sc+scl
sc=sc/n
print, "rbar =",rbar
print, "check =",sc
if (rbar.lt.sc) go to 50
print, "insufficient data - use standard test"
stop
50 continue
print, "prior acceptable"
****** solve for lambda & gamma by eq. 3.2.1 of RADC-TR-79-121******
****** step 1. determine values of alpha ******
do 60 j=1,1000
a(j)=0.
do 70 j1=1,n
if (r(j1).ge.j) a(j)=a(j)+1
70 continue
if (a(j).eq.0.0) go to 80
60 continue
80 limj=j-1
****** step 2. determine lambda & gamma by iteration ******
gam=1000
  call solve(n,w,gam,amgm,tok,limj,r,t,a)
if (tok.gt.0.0) go to 90
gaml=gam
wl=w
if(w.lt.0.0)gam=gam/2.0
if(w.gt.0.0)gam=2.0*gam
```

```
100 continue
   call solve(n,w,gam,amgm,tok,limj,r,t,a)
if (tok.gt.0.0) go to 90
****** this extrapolates a new value of gamma ******
gamn=gam-w*(gam-gaml)/(w-wl)
gaml=gam
wl=w
gam=gamn
go to 100
90 print,"lambda = ",amgm
print, "gamma = ",gam
end
subroutine solve(n,w,gam,amgm,tok,limj,r,t,a)
 dimension r(1000),t(1000),a(1000)
save
******this finds the value of w for an assumed gamma ******
****** step 1. find lambda as a function of gamma ******
amgmd=0.0;amgmn=0.0
do 15 i=1,n
amgmd=amgmd+t(i)/(t(i)+gam)
15 continue
do 25 i2=1,n
amgmn=amgmn+r(i2)/(t(i2)+gam)
25 continue
amgm=gam*(amgmn/amgmd)
****** step 2. compute w ******
w2=0.0;wl=0.0
do 35 i3=1,n
w2=w2+alog(1+t(i3)/gam)
35 continue
do 45 i4=1,limj
wl=wl+a(i4)/(amgm+i4-1)
45 continue
w=wl-w2
****** step 3. test for w = 0 ******
if (w.gt.0.000001.or.w.lt.-0.000001) go to 16
tok=1.0
return
16 tok=-1.0
return
end
```

5. STRUCTURING TEST PLANS

Test plans are designed to permit accept/reject decisions satisfying designated producer's and consumer's risks. For each curve an operating characteristic curve (see Figure 3) can be drawn. Under the assumed exponential failure distribution the curve can be drawn from the expression:

$$P(A|\theta) = \sum_{r=0}^{r^*} \frac{e^{-T/\theta}(T/\theta)^r}{r!} \tag{5.1}$$

where $P(A|\theta)$ is the probability of acceptance for a given MTBF (θ)

   T is the total test time (the sum of the test times on each sample)

   r^* is the number of failures allowed for acceptance $(r^* + 1 = \text{reject})$

This is a Poisson distribution and for classical test plans, θ is set to θ_1 to compute the consumers risk. The producer's risk is computed by setting θ to θ_0 and subtracting the results from 1. (Probability of rejection = 1 - probability of acceptance). Synthesis of the test involves setting the probability to a desired value, and for various values of r^*, solving for T/θ. Since θ is fixed, this procedure gives a series of tests defined by pairs of r^* and T values yielding the same risk. To provide a test satisfying desired producer's and consumer's risks, two series of tests are produced, one for the desired producer's risk and one for the desired consumer's risk. A pair of r^* and T values in one series which matches (or is satisfactorily close to) a pair in the other series, defines a fixed test that satisfies both risks. MIL-STD-781 provides a tabulation of classical fixed time tests for various risks (as well as sequential tests, which are more sophisticated and permit earlier decisions than fixed time tests for the same risks).

An analogous procedure is used to structure Bayesian fixed time tests. Of necessity, it becomes much more complicated. For example, equation 3-7 gives the posterior consumer's risk as:

$$\beta^* = \frac{\int_0^{\theta_1} P(A|\theta)g(\theta)d\theta}{\int_0^{\infty} P(A|\theta)g(\theta)d\theta} \qquad (5.2)$$

substituting equation 5.1 for $P(A|\theta)$ and 4.1 for $g'(\theta)$ we get:

$$\beta^* = \frac{\int_0^{\theta_1}\left\{\sum_{r=0}^{r^*}\frac{e^{-T/\theta}(T/\theta)^r}{r!}\right\}\frac{\gamma^\lambda}{\Gamma(\lambda)}\theta^{-(\lambda+1)}e^{-\gamma/\theta}d\theta}{\int_0^{\infty}\left\{\sum_{r=0}^{r^*}\frac{e^{-T/\theta}(T/\theta)^r}{r!}\right\}\frac{\gamma^\lambda}{\Gamma(\lambda)}\theta^{-(\lambda+1)}e^{-\gamma/\theta}d\theta} \qquad (5.3)$$

This formidable expression is converted to fixed time test plans (test length T*, accept with r* or less failures) by the following process:

Step 1 - The following normalizing functions are substituted into the risk equations:

$$T^* = T/\theta_0$$

$$\gamma^* = \gamma/\theta_0$$

$$K = \theta_0/\theta_1$$

$$\phi = \theta/\theta_0$$

Equation 5.3 then becomes:

$$\beta^* = \frac{\int_0^{1/K}\left\{\sum_{r=0}^{r^*}\frac{e^{-T^*/\phi}(T^*/\phi)^r}{r!}\right\}\frac{\gamma^{*\lambda}}{\Gamma(\lambda)}\phi^{-(\lambda+1)}e^{-\gamma^*/\phi}d\phi}{\int_0^{\infty}\left\{\sum_{r=0}^{r^*}\frac{e^{-T^*/\phi}(T^*/\phi)^r}{r!}\right\}\frac{\gamma^{*\lambda}}{\Gamma(\lambda)}\phi^{-(\lambda+1)}e^{-\gamma^*/\phi}d\phi} \qquad (5.4)$$
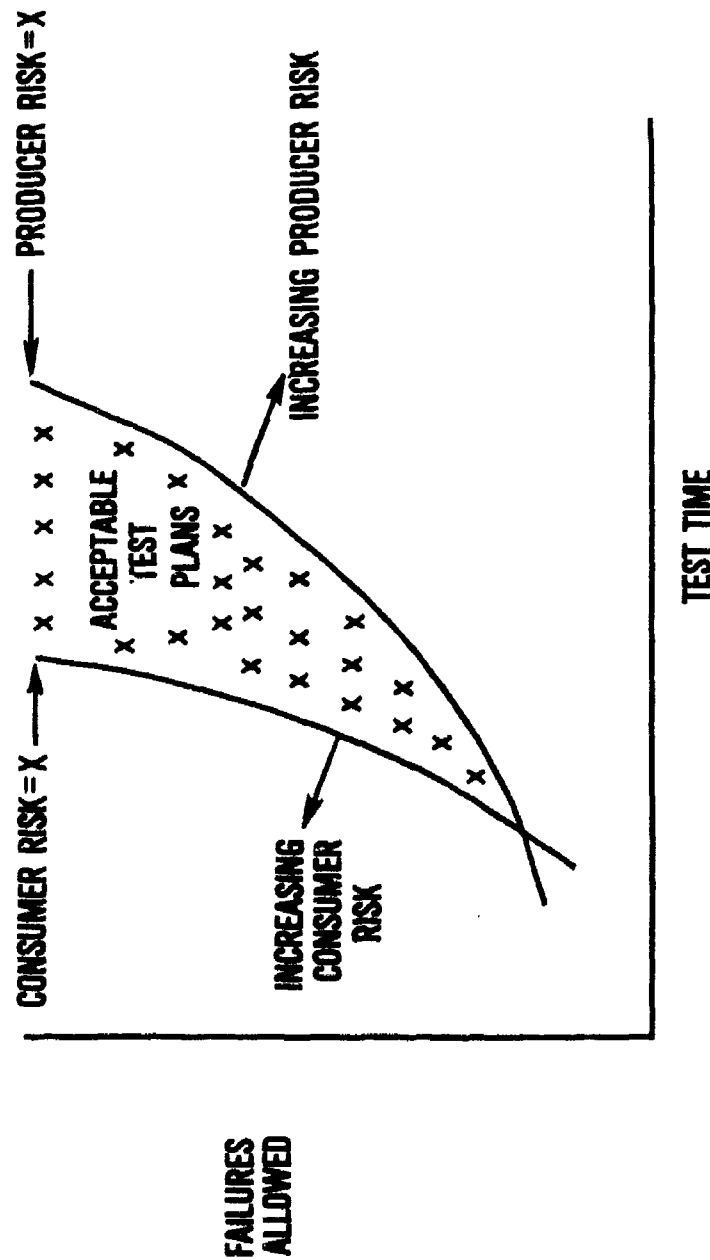
DETERMINING ACCEPTABLE TEST PLANS

FIGURE 6

Step 2 - since γ^*, λ, K, ϕ, and β^* are known, acceptable values of T^* and r^* can be found, if a solution exists. Equation 5.3 computes a consumer's risk,, similar solutions can be found for the desired producer's risk. Figure 6 shows a generalized plot relating r^* and T^* for an arbitrary consumer's risk and a producer's risk. The point at which the two curves cross is the test plan satisfying both risks. This may be found by solving simultaneously the equation for producer's and consumer's risk.

An obvious problem with the preceding approach is that the two curves will rarely cross at an integral number for $\bar{r}^*$. (For example, what does one do with an accept number of 1.76 failures?). One solution is to select the first integer reached on the left hand curve above the intersection. This gives a test plan with the desired consumer's risk but a lower than established producer's risk. In compensation, it provides the shortest possible test time not exceeding either established risk. Plots of risk can be obtained from a computer program presented in RADC-TR-76-294 Vol IV. However, to simplify applications of the procedure, RADC-TR-78-241 "Design of Reliability Test Plans Based Upon Prior Distribution" provides tabulations of test plans, just as MIL-STD-781 tabulates classical fixed time tests.

Test plans are given for the following combination of risk and discrimination ratios (i.e., θ_0/θ_1).

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1. $\bar{\alpha} = \beta^* = 0.10 \quad K = 1.5$

2. $\bar{\alpha} = \beta^* = 0.10 \quad K = 2.0$

3. $\bar{\alpha} = \beta^* = 0.10 \quad K = 3.0$

4. $\bar{\alpha} = \beta^* = 0.15 \quad K = 1.5$

5. $\bar{\alpha} = \beta^* = 0.15 \quad K = 2.0$

6. $\bar{\alpha} = \beta^* = 0.15 \quad K = 3.0$

7. $\bar{\alpha} = \beta^* = 0.20 \quad K = 1.5$

8. $\bar{\alpha} = \beta^* = 0.20 \quad K = 2.0$

9. $\bar{\alpha} = \beta^* = 0.20 \quad K = 3.0$

10. $\alpha^* = \beta^* = 0.10 \quad K = 1.5$

11. $\alpha^* = \beta^* = 0.10 \quad K = 2.0$

12. $\alpha^* = \beta^* = 0.10 \quad K = 3.0$

13. $\alpha^* = \beta^* = 0.15 \quad K = 1.5$

14. $\alpha^* = \beta^* = 0.15 \quad K = 2.0$

15. $\alpha^* = \beta^* = 0.15 \quad K = 3.0$

16. $\alpha^* = \beta^* = 0.20 \quad K = 1.5$

17. $\alpha^* = \beta^* = 0.20 \quad K = 2.0$

18. $\alpha^* = \beta^* = 0.20 \quad K = 3.0$

19. $\bar{\alpha} = \bar{\beta} = 0.10 \quad K = 1.5$

20. $\bar{\alpha} = \bar{\beta} = 0.10 \quad K = 2.0$

21. $\bar{\alpha} = \bar{\beta} = 0.15 \quad K = 1.5$

22. $\bar{\alpha} = \bar{\beta} = 0.15 \quad K = 2.0$

23. $\bar{\alpha} = \bar{\beta} = 0.20 \quad K = 1.5$

24. $\bar{\alpha} = \bar{\beta} = 0.20 \quad K = 2.0$

Unlike MIL-STD-781, each test plan in RADC-TR-78-241 is not one fixed set of values for r^* and t^*. Rather, it is a table of values for different combinations of γ^* and λ which each add another dimension to the problem. Appendix I provides a reprint of RADC-TR-78-241 table 6.20 for $\bar{\alpha} = \bar{\beta} = 0.10$, $K = 2.0$.

These tables will make test selection easy, but present some problems. First, the case of interest may not be among those tabulated. (e.g., the tables do not include any cases where the producer's risk is the overall probability of rejection, nor are discrimination ratios greater than two used for $\bar{\alpha}\ \beta$ test plans). However, the tables probably represent a useful range, since one can always use one of the type of risks presented and discrimination ratios greater than two are philosophically questionable. (If θ_0 represents the achievable state of the art, why should θ_1 be set at less than half that value?)

Another problem with the tables is the r^*, t^* entries of 0,0.00 representing accept without testing and *** representing reject without testing. Neither is really desirable. These can be overcome by designating a minimum test to be used when the prior is favorable enough for a 0,0.00 entry and reverting to a designated classical test when the prior is unfavorable enough for a *** entry. Reversion to a classical test is also a possible alternative when the Bayesian test would be longer than an acceptable classical test.

The final, and most serious problem, is the possibility that the prior data will extend beyond the table limits, (i.e., $\lambda > 4.0$ or $\gamma^* > 3.45$). In this situation, one could revert back to the computer programs of RADC-TR-76-294 or to the original formulas (in Section 3). There is a simpler way, however, which is much easier, but which may horrify statistical purists.

The method basically converts a prior distribution outside the table limits to one with the same mean within the table limits.

The mean of the prior is

$$\mu = \frac{\gamma}{\lambda - 1}$$

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for two distributions to have the same mean

$$\frac{\gamma_1^*}{\lambda_1 - 1} = \frac{\gamma_2^*}{\lambda_2 - 1} \qquad (5.6)$$

(changing γ to γ^* does not affect the equations). Hence, to fit an equation with λ outside the table (γ inside) we simply set γ_1^*, λ_1 equal to the prior data, set λ_2 equal to 4.0, and solve for γ_2^*.

$$\gamma_2^* = \frac{\gamma_1^* \ (4.0-1)}{\lambda_1 - 1} \qquad (5.7)$$

γ_2^* and λ_2 are used with the tables to define the test.

If γ_1^* were outside the table (λ inside) we would set γ_2^* to 3.45 and solve for λ_2 by:

$$\lambda_2 = 1 + \frac{3.45 \ (\lambda_1 - 1)}{\gamma_1^*} \qquad (5.8)$$

If both γ_1^* and λ_1 were outside the table, whichever equation provided values of γ^* and λ within the table would be used.

The effects of the conversion is to change the distribution to one with a wider variance. The median and mode would be shifted downward. Hence, the approach is somewhat conservative, which is the preferred way to err when substituting a Bayesian approach for a classical.

In summary: given the prior, one can derive a test plan from the risk equations in Section 3. This is automated by the computer program of RADC-TR-76-294, Vol IV, which produced the tests tabulated in RADC-TR-78-241. Using the tables is the simplest approach, though they may require facing the problems discussed above.

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6. PUTTING IT ALL TOGETHER:

Applying the preceding discussion to a practical situation starts with the determination that a Bayesian test is feasible (e.g., for production sampling or for testing a commercial unit with available field data). If the test is feasible the manager must determine what type of risks and what risk value he desires. He must also define some default conditions. These are:

(a) A classical test plan to be used if the prior is inadequate, is so unfavorable as to indicate reject without testing, or results in a test longer than an acceptable classical test.

(b) A minimum test to be used if the prior is so favorable as to indicate accept without testing.

(c) A requirement for corrective action and retest if the test is failed. This is a necessity for all test plans. A unique twist in Bayes testing would be the requirement for the first retest after corrective action to be a classical test which would then become the prior for any further tests. The previous prior would be discarded since the corrective action should change the equipment.

These determinations are then put into the contract provisions, which should also require the customer's approval of all data used to structure the prior.

In implementation, the data is first analyzed for sufficiency. If not sufficient, the classical test is used. If sufficient, the λ and γ are obtained and the test plan generated or extracted from the appropriate tables. If the tables are used and λ, γ or both are outside the table limits, the prior is adjusted in accordance with equations 5.7 and 5.8. If the prior is extremely favorable, the minimum test is required. If extremely unfavorable, the

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classical test is mandated. The test is then performed. If successful the test data is added to the previous data and a new prior created for the next test, if any. If the test fails, corrective action is performed and a retest, using the classical test, performed. The results of the classical test becomes the prior for the next test, if any.

Appendix II provides sample requirements for applying Bayesian reliability testing to procurements of commercial equipment. Appendix III provides sample requirements for applying Bayesian techniques to production reliability acceptance testing.

7. EPILOGUE

Bayesian testing has much to recommend it. It is hoped that this report will help to spur its practical application. The author would appreciate any feedback from the readers. Reports of actual implementation would be especially welcome. Criticisms and problems will also be appreciated though probably not enjoyed. Address all comments to

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For those so inclined, Appendix IV provides a listing of RADC studies in Bayesian Reliability Demonstration for more detailed study.

Happy Testing!

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APPENDIX I

The following pages are a reprint of Table 6.20 from RADC-TR-78-241 providing test plans for average risks of 10% and a discrimination ratio of 2.0, over a range of values for the parameters of the prior distribution.

TABLE 6.20

Test Plans for $\lambda = 0.05(0.05)4.00$, $\gamma^* = 1.75(0.05)3.45$ $(\bar{\alpha} = \bar{\beta} = 0.10,\ K = 2.0)$

GAMMA STAR	LAMBDA 0.05		0.10		0.15		0.20		0.25		0.30		0.35		0.40		0.45		0.50	
	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|
0.00	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**
0.05	0.496	01	0.482	01	0.469	01	0.457	01	0.445	01	0.432	01	0.422	01	0.404	01	0.743	01	0.721	01
0.10	0.576	01	0.567	01	0.557	01	0.545	01	0.536	01	0.538	01	0.948	01	0.928	01	0.909	01	0.893	01
0.15	0.629	01	0.619	01	0.609	01	0.602	01	0.595	01	1.057	01	1.043	01	1.028	01	1.014	01	1.000	01
0.20	0.667	01	0.659	01	0.652	01	0.645	01	0.645	01	1.124	01	1.114	01	1.100	01	1.090	01	1.076	01
0.25	0.698	01	0.690	01	0.686	01	0.679	01	1.190	01	1.181	01	1.167	01	1.157	01	1.147	01	1.138	01
0.30	0.724	01	0.719	01	0.712	01	0.705	01	1.271	01	1.228	01	1.214	01	1.203	01	1.195	01	1.186	01
0.35	0.745	01	0.740	01	0.733	01	0.729	01	1.305	01	1.262	01	1.252	01	1.243	01	1.238	01	1.228	01
0.40	0.764	01	0.759	01	0.755	01	0.750	01	1.328	01	1.295	01	1.286	01	1.281	01	1.271	01	1.262	01
0.45	0.781	01	0.776	01	0.771	01	0.767	01	1.357	01	1.328	01	1.319	01	1.309	01	1.300	01	1.295	01
0.50	0.795	01	0.793	01	0.790	01	0.786	01	1.381	01	1.347	01	1.343	01	1.338	01	1.324	01	1.324	01
0.55	0.809	01	0.805	01	0.805	01	0.800	01	1.400	01	1.374	01	1.366	01	1.362	01	1.352	01	1.347	01
0.60	0.824	01	0.819	01	0.814	01	0.812	01	1.419	01	1.395	01	1.385	01	1.381	01	1.376	01	1.371	01
0.65	0.833	01	0.831	01	0.828	01	0.824	01	1.438	01	1.410	01	1.409	01	1.405	01	1.395	01	1.390	01
0.70	0.843	01	0.843	01	0.838	01	0.836	01	1.452	01	1.438	01	1.424	01	1.419	01	1.413	01	1.409	01
0.75	0.855	01	0.852	01	0.848	01	0.848	01	1.466	01	1.447	01	1.434	01	1.438	01	1.433	01	1.428	01
0.80	0.862	01	0.862	01	0.857	01	0.857	01	1.843	01	1.462	01	1.457	01	1.452	01	1.447	01	1.443	01
0.85	0.871	01	0.869	01	0.867	01	0.864	01	1.495	01	1.478	01	1.471	01	1.466	01	1.462	01	1.457	01
0.90	0.881	01	0.876	01	0.876	01	0.871	01	1.881	01	1.490	01	1.485	01	1.481	01	1.476	01	1.471	01
0.95	0.886	01	0.886	01	0.881	01	0.881	01	1.881	01	1.500	01	1.500	01	1.495	01	1.490	01	1.485	01
1.00	0.895	01	0.890	01	0.890	01	0.886	01	1.881	01	1.514	01	1.509	01	1.503	02	1.500	01	1.500	01
1.05	0.900	01	0.900	01	0.895	01	0.895	01	1.919	01	1.524	01	1.519	01	1.519	01	1.509	01	1.509	01
1.10	0.905	01	0.905	01	0.905	01	0.900	01	1.919	01	1.538	01	1.533	01	1.528	01	1.514	01	1.519	01
1.15	0.914	01	0.909	01	0.909	01	0.905	01	1.919	01	1.548	01	1.543	01	1.538	01	1.524	01	1.524	01
1.20	0.919	01	0.917	01	0.914	01	0.914	01	1.919	01	1.552	01	1.552	01	1.547	01	1.533	01	1.533	01
1.25	0.924	01	0.924	01	0.924	01	0.919	01	1.938	01	1.562	01	1.557	01	1.557	01	1.543	01	1.543	01
1.30	0.928	01	0.928	01	0.928	01	0.924	01	1.938	01	1.571	01	1.566	01	1.566	01	1.552	01	1.552	01
1.35	0.933	01	0.933	01	0.933	01	0.928	01	1.940	01	1.581	01	1.576	01	1.576	01	1.562	01	1.557	01
1.40	0.938	01	0.938	01	0.938	01	0.933	01	1.943	01	1.585	01	1.585	01	1.581	01	1.571	01	1.566	01
1.45	0.943	01	0.940	01	0.943	01	0.938	01	1.948	01	1.595	01	1.590	01	1.590	01	1.576	01	1.576	01
1.50	0.948	01	0.943	01	0.948	01	0.943	01	1.940	01	1.600	01	1.600	01	1.598	01	1.585	01	1.585	01
1.55	0.952	01	0.948	01	0.952	01	0.948	01	1.943	01	1.609	01	1.604	01	1.608	01	1.595	01	1.590	01
1.60	0.952	01	0.952	01	0.957	01	0.950	01	1.948	01	1.619	01	1.614	01	1.609	01	1.600	01	1.600	01
1.65	0.957	01	0.957	01	0.957	01	0.952	01	0.952	01	1.619	01	1.619	01	1.618	01	1.608	01	1.608	01
1.70	0.962	01	0.962	01	0.957	01	0.957	01	0.955	01	0.957	01	1.624	01	1.628	01	1.619	01	1.619	01

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TABLE 6.20 (Continued)

GAMMA STAR	LAMBDA 0.05			0.10			0.15			0.20			0.25			0.30			0.35			0.40			0.45			0.50	
	P*	T*	R*	P*	T*	R*	P*	T*	R*	P*	T*	R*	P*	T*	R*	P*	T*	R*	P*	T*	R*	P*	T*	R*	P*	T*	R*	R*	T*
1.75	0	0.967	0	01	0.962	01	01	0.962	01	01	0.962	01	01	0.962	01	01	0.957	01	01	1.628	01	01	1.628	01	01	1.624	01	0	1.624
1.80	01	0.967	01	01	0.967	01	01	0.967	01	01	0.967	01	01	0.962	01	01	0.957	01	01	1.633	01	01	1.633	01	01	1.633	01	0	1.628
1.85	01	0.971	01	01	0.971	01	01	0.969	01	01	0.967	01	01	0.967	01	01	0.957	01	01	1.643	01	01	1.638	01	01	1.638	01	0	1.633
1.90	01	0.976	01	01	0.971	01	01	0.971	01	01	0.971	01	01	0.971	01	01	0.957	01	01	1.643	01	01	1.643	01	01	1.643	01	0	1.638
1.95	01	0.981	01	01	0.976	01	01	0.976	01	01	0.976	01	01	0.971	01	01	0.957	01	01	1.652	01	01	1.647	01	01	1.647	01	0	1.643
2.00	01	0.981	01	01	0.976	01	01	0.976	01	01	0.976	01	01	0.976	01	01	0.957	01	01	1.657	01	01	1.652	01	01	1.652	01	0	1.652
2.05	01	0.981	01	01	0.981	01	01	0.981	01	01	0.981	01	01	0.981	01	01	0.957	01	01	1.662	01	01	1.657	01	01	1.657	01	0	1.652
2.10	01	0.986	01	01	0.986	01	01	0.986	01	01	0.981	01	01	0.981	01	01	0.995	01	01	1.662	01	01	1.662	01	01	1.662	01	0	1.662
2.15	01	0.986	01	01	0.986	01	01	0.986	01	01	0.986	01	01	0.981	01	01	0.995	01	01	1.671	01	01	1.666	01	01	1.666	01	0	1.662
2.20	01	0.986	01	01	0.986	01	01	0.990	01	01	0.986	01	01	0.986	01	01	0.995	01	01	1.676	01	01	1.671	01	01	1.671	01	0	1.666
2.25	01	0.990	01	01	0.990	01	01	0.990	01	01	0.990	01	01	0.986	01	01	0.990	01	01	1.676	01	01	1.676	01	01	1.671	01	0	1.671
2.30	01	0.995	01	01	0.995	01	01	0.995	01	01	0.993	01	01	0.990	01	01	0.995	01	01	1.681	01	01	1.681	01	01	1.676	01	0	1.676
2.35	01	0.995	01	01	0.995	01	01	0.995	01	01	0.995	01	01	0.995	01	01	1.000	01	01	1.685	01	01	1.685	01	01	1.681	01	0	1.681
2.40	01	0.997	01	01	1.000	01	01	1.000	01	01	1.000	01	01	0.995	01	01	1.000	01	01	1.690	01	01	1.690	01	01	1.685	01	0	1.685
2.45	01	1.000	01	01	1.000	01	01	1.000	01	01	1.000	01	01	1.000	01	01	1.000	01	01	0.995	01	01	1.695	01	01	1.690	01	0	1.690
2.50	01	1.005	01	01	1.005	01	01	1.002	01	01	1.005	01	01	1.005	01	01	1.000	01	01	0.995	01	01	1.698	01	01	1.690	01	0	1.690
2.55	01	1.009	01	01	1.005	01	01	1.005	01	01	1.005	01	01	1.005	01	01	1.005	01	01	0.995	01	01	1.700	01	01	1.695	01	0	1.695
2.60	01	1.009	01	01	1.009	01	01	1.005	01	01	1.009	01	01	1.005	01	01	1.005	01	01	0.995	01	01	1.700	01	01	1.700	01	0	1.700
2.65	01	1.014	01	01	1.009	01	01	1.009	01	01	1.009	01	01	1.009	01	01	1.008	01	01	0.995	01	01	1.704	01	01	1.700	01	0	1.700
2.70	01	1.014	01	01	1.014	01	01	1.009	01	01	1.014	01	01	1.014	01	01	1.009	01	01	1.009	01	01	1.709	01	01	1.704	01	0	1.704
2.75	01	1.019	01	01	1.014	01	01	1.014	01	01	1.014	01	01	1.014	01	01	1.012	01	01	1.033	01	01	1.709	01	01	1.709	01	0	1.709
2.80	01	1.019	01	01	1.017	01	01	1.014	01	01	1.014	01	01	1.014	01	01	1.014	01	01	1.014	01	01	1.714	01	01	1.709	01	0	1.709
2.85	01	1.021	01	01	1.019	01	01	1.019	01	01	1.017	01	01	1.019	01	01	1.014	01	01	1.019	01	01	1.719	01	01	1.714	01	0	1.714
2.90	01	1.024	01	01	1.024	01	01	1.019	01	01	1.019	01	01	1.019	01	01	1.019	01	01	1.021	01	01	1.719	01	01	1.714	01	0	1.714
2.95	01	1.024	01	01	1.024	01	01	1.024	01	01	1.024	01	01	1.024	01	01	1.019	01	01	1.024	01	01	1.719	01	01	1.719	01	0	1.718
3.00	01	1.026	01	01	1.028	01	01	1.024	01	01	1.024	01	01	1.024	01	01	1.019	01	01	1.024	01	01	0.998	01	01	1.724	01	0	1.724
3.05	01	1.028	01	01	1.028	01	01	1.024	01	01	1.024	01	01	1.024	01	01	1.024	01	01	1.024	01	01	0.995	01	01	1.724	01	0	1.724
3.10	01	1.028	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.024	01	01	1.028	01	01	1.033	01	01	1.728	01	0	1.728
3.15	01	1.033	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.033	01	01	1.733	01	0	1.728
3.20	01	1.033	01	01	1.031	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.028	01	01	1.024	01	01	1.033	01	01	1.733	01	0	1.733
3.25	01	1.038	01	01	1.033	01	01	1.033	01	01	1.028	01	01	1.033	01	01	1.028	01	01	1.028	01	01	1.033	01	01	1.738	01	0	1.738
3.30	01	1.038	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.028	01	01	1.028	01	01	1.033	01	01	1.738	01	0	1.738
3.35	01	1.038	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.028	01	01	1.033	01	01	1.743	01	0	1.738
3.40	01	1.038	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.028	01	01	1.033	01	01	1.743	01	0	1.743
3.45	01	1.039	01	01	1.036	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.033	01	01	1.747	01	0	1.743

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TABLE 6.20 (Continued)

GAMMA STAR	λ=0.55 T*	λ=0.55 |R*|	λ=0.60 T*	λ=0.60 |R*|	λ=0.65 T*	λ=0.65 |R*|	λ=0.70 T*	λ=0.70 |R*|	λ=0.75 T*	λ=0.75 |R*|	λ=0.80 T*	λ=0.80 |R*|	λ=0.85 T*	λ=0.85 |R*|	λ=0.90 T*	λ=0.90 |R*|	λ=0.95 T*	λ=0.95 |R*|	λ=1.00 T*	λ=1.00 |R*|
0.00	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**
0.05	0.700	1	0.676	1	0.652	1	0.629	1	0.605	1	0.581	1	0.560	1	0.538	1	0.514	1	0.492	1
0.10	0.876	1	0.857	1	0.838	1	0.819	1	0.800	1	0.781	1	0.762	1	0.743	1	0.724	1	0.705	1
0.15	0.986	1	0.967	1	0.952	1	0.938	1	0.924	1	0.905	1	0.890	1	0.871	1	0.857	1	0.840	1
0.20	1.062	1	1.047	1	1.038	1	1.024	1	1.009	1	0.995	1	0.981	1	0.967	1	0.952	1	0.938	1
0.25	1.124	1	1.114	1	1.100	1	1.090	1	1.076	1	1.067	1	1.109	1	1.033	1	1.033	1	1.033	1
0.30	1.176	1	1.167	1	1.152	1	1.143	1	1.147	1	1.109	1	1.533	1	1.557	1	1.071	1	1.824	1
0.35	1.219	1	1.209	1	1.200	1	1.186	1	1.186	1	1.147	1	1.690	1	1.619	1	1.604	1	1.595	1
0.40	1.252	1	1.247	1	1.243	1	1.224	1	1.224	1	1.186	1	1.724	1	1.676	1	1.662	1	1.652	1
0.45	1.286	1	1.281	1	1.262	1	1.262	1	1.757	1	1.747	1	1.776	1	1.724	1	1.714	1	1.700	1
0.50	1.314	1	1.309	1	1.300	1	1.300	1	1.795	1	1.785	1	1.814	1	1.766	1	1.757	1	1.747	1
0.55	1.343	1	1.335	1	1.338	1	1.804	1	1.833	1	1.823	1	1.847	1	1.804	1	1.795	1	1.785	1
0.60	1.366	1	1.376	1	1.338	1	1.871	1	1.866	1	1.857	1	1.881	1	1.838	1	1.833	1	1.823	1
0.65	1.385	1	1.376	1	1.376	1	1.904	1	1.895	1	1.885	1	1.909	1	1.871	1	1.862	1	1.852	1
0.70	1.405	1	1.414	1	1.376	1	1.928	1	1.919	1	1.914	1	1.933	1	1.900	1	1.890	1	1.881	1
0.75	1.424	1	1.414	1	1.414	1	1.952	1	1.947	1	1.936	1	1.957	1	1.923	1	1.919	1	1.909	1
0.80	1.438	1	1.452	1	1.414	1	1.976	1	1.966	1	1.962	1	1.976	1	1.947	1	1.942	1	1.938	1
0.85	1.452	1	1.452	1	1.452	1	1.995	1	1.990	1	1.985	1	2.000	1	1.971	1	1.966	1	1.957	1
0.90	1.466	1	1.490	1	2.004	1	2.014	1	2.009	1	2.008	1	2.014	1	1.995	1	1.985	1	1.981	1
0.95	1.481	1	1.490	1	1.490	1	2.033	1	2.028	1	2.029	1	2.033	1	2.018	1	2.004	1	2.000	1
1.00	1.495	1	1.509	1	1.490	1	2.052	1	2.047	1	2.042	1	2.052	1	2.028	1	2.021	1	2.019	1
1.05	1.505	1	1.526	1	1.528	1	2.066	1	2.062	1	2.057	1	2.066	1	2.047	1	2.042	1	2.033	1
1.10	1.519	1	1.524	1	1.528	1	2.081	1	2.076	1	2.074	1	2.081	1	2.062	1	2.057	1	2.052	1
1.15	1.528	1	1.533	1	1.566	1	1.490	1	2.090	1	2.085	1	2.095	1	2.076	1	2.071	1	2.066	1
1.20	1.547	1	1.543	1	1.566	1	2.109	1	2.104	1	2.100	1	2.109	1	2.090	1	2.085	1	2.081	1
1.25	1.557	1	1.552	1	1.565	1	2.123	1	2.119	1	2.118	1	2.123	1	2.104	1	2.100	1	2.095	1
1.30	1.566	1	1.562	1	1.604	1	1.528	1	2.128	1	2.128	1	2.133	1	2.119	1	2.114	1	2.109	1
1.35	1.571	1	1.571	1	1.595	1	1.528	1	2.142	1	2.138	1	2.147	1	2.128	1	2.128	1	2.123	1
1.40	1.576	1	1.576	1	1.604	1	1.566	1	2.152	1	2.147	1	2.157	1	2.142	1	2.138	1	2.133	1
1.45	1.581	1	1.585	1	1.609	1	1.566	1	2.166	1	2.162	1	2.166	1	2.152	1	2.147	1	2.147	1
1.50	1.585	1	1.595	1	—	1	1.566	1	2.176	1	2.171	1	2.176	1	2.166	1	2.157	1	2.157	1
1.55	1.595	1	1.600	1	—	1	1.566	1	1.566	1	2.181	1	2.185	1	2.176	1	2.171	1	2.166	1
1.60	1.600	1	1.604	1	—	1	1.604	1	2.204	1	2.190	1	2.195	1	2.185	1	2.181	1	2.176	1
1.65	1.604	1	1.614	1	—	1	1.604	1	2.204	1	2.200	1	2.195	1	2.195	1	2.190	1	2.185	1
1.70	1.614	1	—	1	—	1	1.604	1	2.209	1	2.208	1	2.204	1	2.204	1	2.200	1	2.195	1

TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	0.55		0.60		0.65		0.70		0.75		0.80		0.85		0.90		0.95		1.00	
	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*
1.75	11	1.624	11	1.619	11	1.614	11	1.604	11	1.604	11	2.214	11	2.214	11	2.209	11	2.204	11	2.204
1.80	11	1.628	11	1.624	11	1.624	11	1.643	11	1.643	11	2.228	11	2.223	11	2.219	11	2.214	11	2.214
1.85	11	1.633	11	1.633	11	1.628	11	1.643	11	1.643	11	2.233	11	2.228	11	2.223	11	2.223	11	2.219
1.90	11	1.638	11	1.633	11	1.638	11	1.633	11	1.643	11	2.239	11	2.238	11	2.233	11	2.233	11	2.228
1.95	11	1.643	11	1.643	11	1.643	11	1.638	11	1.643	11	1.604	11	2.242	11	2.242	11	2.238	11	2.233
2.00	11	1.647	11	1.647	11	1.643	11	1.643	11	1.643	11	1.643	11	2.252	11	2.247	11	2.242	11	2.242
2.05	11	1.652	11	1.652	11	1.647	11	1.647	11	1.643	11	1.643	11	2.257	11	2.252	11	2.252	11	2.252
2.10	11	1.657	11	1.657	11	1.652	11	1.652	11	1.643	11	1.643	11	2.261	11	2.261	11	2.261	11	2.257
2.15	11	1.662	11	1.662	11	1.657	11	1.657	11	1.681	11	1.643	11	2.271	11	2.266	11	2.266	11	2.261
2.20	11	1.666	11	1.666	11	1.662	11	1.662	11	1.681	11	1.643	11	2.276	11	2.271	11	2.271	11	2.271
2.25	11	1.671	11	1.671	11	1.666	11	1.666	11	1.662	11	1.643	11	1.643	11	2.281	11	2.281	11	2.276
2.30	11	1.676	11	1.676	11	1.671	11	1.671	11	1.671	11	1.643	11	1.643	11	2.285	11	2.281	11	2.281
2.35	11	1.681	11	1.681	11	1.676	11	1.676	11	1.676	11	1.681	11	1.643	11	2.290	11	2.290	11	2.290
2.40	11	1.681	11	1.685	11	1.681	11	1.681	11	1.681	11	1.681	11	1.681	11	1.643	11	2.295	11	2.290
2.45	11	1.685	11	1.690	11	1.681	11	1.681	11	1.685	11	1.681	11	1.681	11	2.300	11	2.300	11	2.300
2.50	11	1.690	11	1.690	11	1.685	11	1.685	11	1.690	11	1.685	11	1.681	11	2.309	11	2.304	11	2.304
2.55	11	1.695	11	1.695	11	1.690	11	1.690	11	1.690	11	1.690	11	1.681	11	1.681	11	2.314	11	2.309
2.60	11	1.700	11	1.700	11	1.695	11	1.690	11	1.695	11	1.695	11	1.681	11	1.681	11	2.319	11	2.314
2.65	11	1.704	11	1.700	11	1.700	11	1.695	11	1.700	11	1.700	11	1.719	11	1.719	11	2.323	11	2.319
2.70	11	1.709	11	1.704	11	1.700	11	1.700	11	1.704	11	1.700	11	1.719	11	1.719	11	2.328	11	2.323
2.75	11	1.709	11	1.709	11	1.704	11	1.704	11	1.704	11	1.704	11	1.700	11	1.719	11	1.681	11	2.328
2.80	11	1.714	11	1.709	11	1.709	11	1.704	11	1.709	11	1.709	11	1.704	11	1.719	11	1.719	11	2.328
2.85	11	1.714	11	1.714	11	1.709	11	1.709	11	1.709	11	1.709	11	1.709	11	1.714	11	1.719	11	2.338
2.90	11	1.716	11	1.716	11	1.714	11	1.714	11	1.714	11	1.709	11	1.709	11	1.716	11	1.719	11	2.338
2.95	11	1.718	11	1.718	11	1.714	11	1.714	11	1.718	11	1.716	11	1.714	11	1.718	11	1.719	11	2.342
3.00	11	1.724	11	1.724	11	1.716	11	1.716	11	1.724	11	1.716	11	1.716	11	1.724	11	1.719	11	1.681
3.05	11	1.728	11	1.728	11	1.724	11	1.718	11	1.728	11	1.718	11	1.718	11	1.728	11	1.719	11	1.719
3.10	11	1.728	11	1.728	11	1.728	11	1.724	11	1.728	11	1.728	11	1.724	11	1.728	11	1.728	11	1.719
3.15	11	1.733	11	1.728	11	1.733	11	1.728	11	1.733	11	1.728	11	1.728	11	1.733	11	1.728	11	1.719
3.20	11	1.738	11	1.733	11	1.733	11	1.728	11	1.738	11	1.733	11	1.728	11	1.738	11	1.733	11	1.719
3.25	11	1.738	11	1.738	11	1.738	11	1.733	11	1.738	11	1.738	11	1.733	11	1.738	11	1.738	11	1.719
3.30	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.719
3.35	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.728	11	1.728
3.40	11	1.743	11	1.738	11	1.743	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.733	11	1.757
3.45	11	1.743	11	1.743	11	1.743	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.738	11	1.757

TABLE 6.20 (Continued)

GAMMA STAR		LAMBDA																		
		1.05		1.10		1.15		1.20		1.25		1.30		1.35		1.40		1.45		1.50
	|R|	T*	|R|	T*	|R|	T*	|R|	T*	|R|	T*	|R|	T*	|R|	T*	|R|	T*	|R|	T*	|R|	T*
0.00	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****	**	****
0.05	1	0.673	1	0.511	1	0.430	1	0.203	1	0.203	1	0.188	1	0.188	1	0.160	1	0.172	1	0.160
0.10	1	0.686	1	0.667	1	0.648	1	0.629	1	0.612	1	0.503	1	0.576	1	0.557	1	0.540	1	0.524
0.15	1	0.826	1	0.809	1	0.790	1	0.776	1	0.757	1	0.740	1	0.724	1	0.708	1	0.690	1	0.676
0.20	1	0.924	1	0.909	1	0.895	1	0.878	1	0.867	1	0.850	1	0.833	1	0.819	1	0.803	1	0.790
0.25	1	0.994	1	0.995	1	0.957	1	0.957	1	0.938	1	0.918	1	0.919	1	0.900	1	0.881	1	0.881
0.30	2	1.509	2	1.495	2	1.481	2	1.462	2	1.447	2	1.428	2	1.414	2	1.395	2	1.381	2	1.362
0.35	2	1.581	2	1.565	2	1.552	2	1.538	2	1.519	2	1.509	2	1.490	2	1.476	2	1.462	2	1.443
0.40	2	1.638	2	1.624	2	1.614	2	1.600	2	1.585	2	1.571	2	1.557	2	1.549	2	1.528	2	1.519
0.45	2	1.690	2	1.681	2	1.666	2	1.652	2	1.643	2	1.681	2	1.614	2	1.604	2	1.590	2	1.576
0.50	2	1.738	2	1.724	2	1.714	2	1.700	2	1.690	2	1.726	2	1.666	2	1.657	2	1.643	2	1.633
0.55	2	1.776	2	1.766	2	1.757	2	1.747	2	1.733	2	1.766	2	1.714	2	1.700	2	1.690	2	1.681
0.60	2	1.814	2	1.804	2	1.795	2	1.785	2	1.776	2	1.800	2	1.757	2	1.743	2	1.733	2	1.724
0.65	2	1.843	2	1.838	2	1.828	2	1.819	2	1.843	2	1.838	2	1.790	2	1.795	2	1.795	2	1.757
0.70	2	1.876	2	1.866	2	1.857	2	1.852	2	1.869	2	1.871	2	1.833	2	1.833	2	1.795	2	1.795
0.75	2	1.900	2	1.895	2	1.885	2	1.881	2	1.909	2	1.909	2	1.871	2	1.871	2	1.833	2	1.833
0.80	2	1.928	2	1.919	2	1.914	2	1.904	2	1.909	2	1.909	2	1.871	2	1.871	2	1.871	2	1.833
0.85	2	1.952	2	1.947	2	1.938	2	1.928	2	1.947	2	1.947	2	1.909	2	1.871	2	1.871	2	1.871
0.90	2	1.976	2	1.966	2	1.962	2	1.966	2	1.947	2	1.947	2	1.947	2	1.909	2	2.438	2	1.871
0.95	2	1.995	2	1.985	2	1.981	2	1.985	2	1.985	2	1.985	2	1.947	2	1.947	2	1.909	2	2.428
1.00	2	2.018	2	2.004	2	2.023	2	2.004	2	2.023	2	2.023	2	1.947	2	1.947	2	2.490	2	2.457
1.05	2	2.033	2	2.022	2	2.019	2	2.023	2	2.023	2	2.023	2	1.985	2	2.519	2	2.514	2	2.480
1.10	2	2.047	2	2.042	2	2.052	2	2.023	2	2.023	2	2.062	2	1.985	2	2.547	2	2.538	2	2.509
1.15	2	2.062	2	2.057	2	2.066	2	2.062	2	2.062	2	2.062	2	2.023	2	2.566	2	2.557	2	2.528
1.20	2	2.081	2	2.071	2	2.100	2	2.100	2	2.100	2	2.100	2	2.023	2	2.585	2	2.576	2	2.552
1.25	2	2.090	2	2.085	2	2.095	2	2.100	2	2.100	2	2.100	2	2.062	2	2.604	2	2.595	2	2.576
1.30	2	2.104	2	2.100	2	2.138	2	2.138	2	2.100	2	2.100	2	2.062	2	2.628	2	2.609	2	2.595
1.35	2	2.119	2	2.114	2	2.119	2	2.138	2	2.138	2	2.138	2	2.100	2	2.638	2	2.633	2	2.609
1.40	2	2.128	2	2.128	2	2.133	2	2.176	2	2.176	2	2.176	2	2.100	2	2.652	2	2.652	2	2.628
1.45	2	2.142	2	2.138	2	2.147	2	2.176	2	2.176	2	2.176	2	2.100	2	2.671	2	2.661	2	2.642
1.50	2	2.152	2	2.147	2	2.157	2	2.176	2	2.176	2	2.176	2	2.719	2	2.685	2	2.680	2	2.661
1.55	2	2.162	2	2.157	2	2.166	2	2.176	2	2.176	2	2.138	2	2.138	2	2.699	2	2.695	2	2.676
1.60	2	2.176	2	2.166	2	2.173	2	2.171	2	2.176	2	2.176	2	2.176	2	2.709	2	2.709	2	2.690
1.65	2	2.185	2	2.181	2	2.173	2	2.171	2	2.176	2	2.176	2	2.176	2	2.723	2	2.719	2	2.699
1.70	2	2.195	2	2.185	2	2.185	2	2.195	3	2.176	2	2.176	2	2.176	2	2.739	2	2.733	2	2.728

TABLE 5.20 (Continued)

GAMMA STAR	LAMBDA 1.35		1.10		1.15		1.20		1.25		1.30		1.35		1.40		1.45		1.50	
	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*
1.75	21	2.200	21	2.195	21	2.195	21	2.214	21	2.176	21	2.176	21	2.176	21	2.747	21	2.747	21	2.738
1.80	21	2.209	21	2.204	21	2.204	21	2.209	21	2.214	21	2.176	21	2.176	21	2.757	21	2.757	21	2.752
1.85	21	2.219	21	2.214	21	2.214	21	2.209	21	2.214	21	2.176	21	2.176	21	2.176	21	2.766	21	2.761
1.90	21	2.223	21	2.223	21	2.219	21	2.214	21	2.214	21	2.214	21	2.176	21	2.176	21	2.776	21	2.776
1.95	21	2.233	21	2.233	21	2.228	21	2.223	21	2.233	21	2.214	21	2.214	21	2.790	21	2.176	21	2.785
2.00	21	2.242	21	2.238	21	2.233	21	2.233	21	2.252	21	2.252	21	2.252	21	2.799	21	2.795	21	2.795
2.05	21	2.247	21	2.242	21	2.242	21	2.238	21	2.239	21	2.252	21	2.252	21	2.214	21	2.804	21	2.804
2.10	21	2.252	21	2.252	21	2.252	21	2.247	21	2.242	21	2.252	21	2.252	21	2.252	21	2.814	21	2.814
2.15	21	2.261	21	2.257	21	2.257	21	2.252	21	2.252	21	2.252	21	2.252	21	2.252	21	2.214	21	2.823
2.20	21	2.266	21	2.261	21	2.261	21	2.261	21	2.357	21	2.252	21	2.252	21	2.252	21	2.214	21	2.828
2.25	21	2.271	21	2.271	21	2.271	21	2.266	21	2.261	21	2.252	21	2.252	21	2.252	21	2.252	21	2.838
2.30	21	2.276	21	2.276	21	2.276	21	2.271	21	2.271	21	2.266	21	2.290	21	2.252	21	2.252	21	2.842
2.35	21	2.281	21	2.281	21	2.281	21	2.281	21	2.276	21	2.271	21	2.290	21	2.252	21	2.252	21	2.252
2.40	21	2.285	21	2.290	21	2.285	21	2.285	21	2.281	21	2.285	21	2.290	21	2.252	21	2.252	21	2.252
2.45	21	2.290	21	2.295	21	2.290	21	2.290	21	2.290	21	2.281	21	2.290	21	2.252	21	2.252	21	2.252
2.50	21	2.295	21	2.300	21	2.300	21	2.295	21	2.290	21	2.290	21	2.290	21	2.290	21	2.252	21	2.252
2.55	21	2.300	21	2.304	21	2.300	21	2.300	21	2.300	21	2.295	21	2.295	21	2.290	21	2.290	21	2.290
2.60	21	2.304	21	2.309	21	2.309	21	2.308	21	2.304	21	2.300	21	2.300	21	2.309	21	2.290	21	2.290
2.65	21	2.309	21	2.314	21	2.309	21	2.309	21	2.309	21	2.308	21	2.304	21	2.309	21	2.328	21	2.328
2.70	21	2.314	21	2.319	21	2.319	21	2.314	21	2.314	21	2.309	21	2.309	21	2.309	21	2.328	21	2.328
2.75	21	2.319	21	2.323	21	2.319	21	2.319	21	2.319	21	2.319	21	2.314	21	2.319	21	2.328	21	2.328
2.80	21	2.323	21	2.326	21	2.323	21	2.323	21	2.323	21	2.319	21	2.319	21	2.319	21	2.328	21	2.328
2.85	21	2.327	21	2.333	21	2.328	21	2.326	21	2.326	21	2.323	21	2.323	21	2.323	21	2.323	21	2.328
2.90	21	2.333	21	2.338	21	2.338	21	2.333	21	2.328	21	2.327	21	2.326	21	2.328	21	2.328	21	2.328
2.95	21	2.338	21	2.342	21	2.342	21	2.338	21	2.338	21	2.338	21	2.333	21	2.338	21	2.326	21	2.326
3.00	21	2.342	21	2.347	21	2.347	21	2.342	21	2.338	21	2.338	21	2.338	21	2.338	21	2.333	21	2.347
3.05	21	2.347	21	2.352	21	2.347	21	2.347	21	2.342	21	2.342	21	2.338	21	2.338	21	2.338	21	2.338
3.10	21	2.347	21	2.357	21	2.352	21	2.347	21	2.347	21	2.347	21	2.347	21	2.342	21	2.338	21	2.338
3.15	21	2.352	21	2.357	21	2.357	21	2.352	21	2.352	21	2.347	21	2.347	21	2.347	21	2.342	21	2.342
3.20	21	2.357	21	2.357	21	2.357	21	2.357	21	2.357	21	2.357	21	2.352	21	2.352	21	2.347	21	2.347
3.25	21	2.357	11	2.361	11	2.361	11	2.361	11	2.357	21	2.357	21	2.357	21	2.357	11	2.352	21	2.352
3.30	21	2.361	11	2.366	11	2.366	11	2.366	11	2.361	11	2.361	21	2.357	21	2.357	11	2.357	11	2.338
3.35	11	1.719	11	2.371	11	2.366	11	2.366	11	2.366	11	2.366	11	2.366	11	2.361	11	2.361	11	2.357
3.40	11	1.719	11	2.376	11	2.371	11	2.371	11	2.366	11	2.366	11	2.366	11	2.366	11	2.366	11	2.361
3.45	11	1.719	11	1.719	11	2.376	11	2.376	11	2.376	11	2.371	11	2.371	11	2.366	11	2.366	11	2.366

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TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	1.55			1.60			1.65			1.70			1.75			1.80			1.85			1.90			1.95			2.00		
	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*	R*	T*	P*
0.00	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
0.05	0	0.153	0	0	0.146	0	0	0.141	0	0	0.135	0	0	0.129	0	0	0.124	0	0	0.119	0	0	0.114	0	0	0.110	0	0	0.105	0
0.10	1	0.507	1	1	0.490	1	1	0.475	1	1	0.459	1	1	0.445	1	1	0.430	1	1	0.416	1	1	0.402	1	1	0.389	1	1	0.377	1
0.15	1	0.657	1	1	0.643	1	1	0.626	1	1	0.609	1	1	0.595	1	1	0.581	1	1	0.567	1	1	0.552	1	1	0.538	1	1	0.524	1
0.20	1	0.776	1	1	0.759	1	1	0.745	1	1	0.729	1	1	0.714	1	1	0.700	1	1	0.686	1	1	0.671	1	1	0.657	1	1	0.643	1
0.25	1	0.881	1	1	0.843	1	1	0.843	1	1	0.805	1	1	0.805	1	1	0.805	1	1	0.786	1	1	0.786	1	1	0.757	1	1	0.743	1
0.30	2	1.343	2	2	1.328	2	2	1.309	2	2	1.290	2	2	1.276	2	2	1.257	2	2	1.238	2	2	1.219	2	2	1.205	2	2	1.186	2
0.35	2	1.428	2	2	1.412	2	2	1.395	2	2	1.381	2	2	1.366	2	2	1.347	2	2	1.333	2	2	1.314	2	2	1.300	2	2	1.281	2
0.40	2	1.500	2	2	1.490	2	2	1.471	2	2	1.457	2	2	1.443	2	2	1.428	2	2	1.412	2	2	1.395	2	2	1.381	2	2	1.366	2
0.45	2	1.566	2	2	1.552	2	2	1.538	2	2	1.524	2	2	1.509	2	2	1.495	2	2	1.481	2	2	1.471	2	2	1.452	2	2	1.438	2
0.50	2	1.619	2	2	1.609	2	2	1.595	2	2	1.585	2	2	1.571	2	2	1.557	2	2	1.543	2	2	1.528	2	2	1.519	2	2	1.505	2
0.55	2	1.671	2	2	1.657	2	2	1.647	2	2	1.633	2	2	1.624	2	2	1.609	2	2	1.600	2	2	1.585	2	2	1.576	2	2	1.562	2
0.60	2	1.714	2	2	1.719	2	2	1.700	2	2	1.681	2	2	1.681	2	2	1.643	2	2	1.643	2	2	1.643	2	2	1.643	2	2	1.604	2
0.65	2	1.757	2	2	1.719	2	2	1.719	2	2	1.719	2	2	1.719	2	2	1.719	2	2	1.681	2	2	1.681	2	2	1.681	2	2	1.643	2
0.70	2	1.795	2	2	1.795	2	2	1.757	2	2	1.757	2	2	1.719	2	2	1.719	2	2	1.719	2	2	2.195	2	2	2.228	2	2	2.171	2
0.75	2	1.795	2	2	1.795	2	2	1.795	2	2	2.285	2	2	1.757	2	2	2.261	2	2	2.252	2	2	2.242	2	2	2.271	2	2	2.219	2
0.80	2	1.833	2	2	2.347	2	2	2.338	2	2	2.323	2	2	2.314	2	2	2.304	2	2	2.290	2	2	2.281	2	3	2.309	3	3	2.261	3
0.85	3	2.385	3	3	2.376	3	3	2.366	3	3	2.357	3	3	2.347	3	3	2.338	3	3	2.327	3	3	2.319	3	3	2.347	3	3	2.300	3
0.90	3	2.419	3	3	2.409	3	3	2.404	3	3	2.395	3	3	2.385	3	3	2.376	3	3	2.366	3	3	2.357	3	3	2.376	3	3	2.338	3
0.95	3	2.447	3	3	2.442	3	3	2.433	3	3	2.423	3	3	2.414	3	3	2.404	3	3	2.395	3	3	2.385	3	3	2.404	3	3	2.366	3
1.00	3	2.476	3	3	2.466	3	3	2.461	3	3	2.452	3	3	2.442	3	3	2.433	3	3	2.423	3	3	2.414	3	3	2.433	3	3	2.400	3
1.05	3	2.500	3	3	2.490	3	3	2.485	3	3	2.476	3	3	2.471	3	3	2.461	3	3	2.452	3	3	2.442	3	3	2.461	3	3	2.428	3
1.10	3	2.523	3	3	2.519	3	3	2.509	3	3	2.500	3	3	2.495	3	3	2.485	3	3	2.480	3	3	2.471	3	3	2.490	3	3	2.452	3
1.15	3	2.547	3	3	2.538	3	3	2.528	3	3	2.523	3	3	2.519	3	3	2.509	3	3	2.500	3	3	2.495	3	3	2.509	3	3	2.478	3
1.20	3	2.566	3	3	2.557	3	3	2.552	3	3	2.547	3	3	2.538	3	3	2.535	3	3	2.528	3	3	2.519	3	3	2.533	3	3	2.504	3
1.25	3	2.580	3	3	2.580	3	3	2.576	3	3	2.566	3	3	2.557	3	3	2.557	3	3	2.547	3	3	2.538	3	3	2.557	3	3	2.528	3
1.30	3	2.604	3	3	2.595	3	3	2.595	3	3	2.585	3	3	2.580	3	3	2.576	3	3	2.566	3	3	2.561	3	3	2.576	3	3	2.547	3
1.35	3	2.623	3	3	2.614	3	3	2.609	3	3	2.604	3	3	2.599	3	3	2.595	3	3	2.585	3	3	2.580	3	3	2.595	3	3	2.566	3
1.40	3	2.638	3	3	2.630	3	3	2.628	3	3	2.623	3	3	2.614	3	3	2.609	3	3	2.604	3	3	2.599	3	3	2.609	3	3	2.585	3
1.45	3	2.652	3	3	2.652	3	3	2.642	3	3	2.638	3	3	2.630	3	3	2.628	3	3	2.623	3	3	2.614	3	3	2.628	3	3	2.604	3
1.50	3	2.671	3	3	2.661	3	3	2.661	3	3	2.652	3	3	2.652	3	3	2.642	3	3	2.638	3	3	2.630	3	3	2.642	3	3	2.623	3
1.55	3	2.680	3	3	2.680	3	3	2.671	3	3	2.671	3	3	2.661	3	3	2.661	3	3	2.652	3	3	2.652	3	3	2.661	3	3	2.638	3
1.60	3	2.699	3	3	2.690	3	3	2.690	3	3	2.680	3	3	2.680	3	3	2.671	3	3	2.671	3	3	2.661	3	3	2.671	3	3	2.652	3
1.65	3	2.709	3	3	2.709	3	3	2.699	3	3	2.699	3	3	2.690	3	3	2.690	3	3	2.680	3	3	2.680	3	3	2.690	3	3	2.671	3
1.70	3	2.723	3	3	2.719	3	3	2.714	3	3	2.709	3	3	2.709	3	3	2.698	3	3	2.699	3	3	2.698	3				3	2.709	3

TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	1.55		1.60		1.65		1.70		1.75		1.80		1.85		1.90		1.95		2.00	
	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*
1.75	31	2.738	31	2.728	31	2.728	31	2.723	31	2.719	31	2.714	31	2.709	31	2.704	31	2.699	31	2.695
1.80	31	2.747	31	2.742	31	2.738	31	2.738	31	2.728	31	2.728	31	2.719	31	2.728	31	2.714	31	2.707
1.85	31	2.757	31	2.757	31	2.747	31	2.747	31	2.742	31	2.747	31	2.738	31	2.738	31	2.728	31	2.733
1.90	31	2.766	31	2.766	31	2.761	31	2.757	31	2.757	31	2.747	31	2.747	31	2.742	31	2.738	31	2.747
1.95	31	2.780	31	2.776	31	2.771	31	2.766	31	2.766	31	2.761	31	2.757	31	2.752	31	2.747	31	2.765
2.00	31	2.790	31	2.785	31	2.785	31	2.776	31	2.776	31	2.771	31	2.766	31	2.766	31	2.757	31	2.766
2.05	31	2.799	31	2.795	31	2.795	31	2.785	31	2.785	31	2.780	31	2.776	31	2.776	31	2.771	31	2.776
2.10	31	2.809	31	2.804	31	2.804	31	2.795	31	2.795	31	2.790	31	2.785	31	2.780	31	2.780	31	2.783
2.15	31	2.814	31	2.814	31	2.814	31	2.804	31	2.804	31	2.799	31	2.795	31	2.795	31	2.790	31	2.795
2.20	39	2.823	31	2.823	31	2.818	31	2.814	31	2.814	31	2.809	31	2.804	31	2.804	31	2.799	31	2.804
2.25	31	2.833	31	2.833	31	2.828	31	2.823	31	2.823	31	2.818	31	2.814	31	2.814	31	2.809	31	2.814
2.30	31	2.842	31	2.842	31	2.833	31	2.833	31	2.833	31	2.823	31	2.823	31	2.823	31	2.823	31	2.823
2.35	31	2.852	31	2.847	31	2.842	31	2.842	31	2.838	31	2.833	31	2.833	31	2.828	31	2.833	31	2.833
2.40	31	2.857	31	2.852	31	2.852	31	2.852	31	2.842	31	2.840	31	2.842	31	2.838	31	2.833	31	2.842
2.45	31	2.861	31	2.661	31	2.861	31	2.857	31	2.852	31	2.852	21	2.847	31	2.842	21	2.842	21	2.847
2.50	21	2.252	21	2.871	21	2.866	21	2.861	21	2.861	31	2.861	31	2.861	21	2.852	21	2.852	21	2.852
2.55	31	2.865	31	2.875	31	2.871	31	2.871	21	2.871	31	2.866	31	2.861	31	2.861	31	2.857	21	2.861
2.60	31	2.890	31	2.880	31	2.880	31	2.880	31	2.876	31	2.871	31	2.871	31	2.866	31	2.661	31	2.861
2.65	21	2.290	21	2.890	21	2.890	21	2.890	21	2.660	31	2.880	31	2.875	31	2.671	31	2.871	31	2.871
2.70	21	2.328	21	2.895	21	2.890	21	2.899	21	2.890	31	2.885	21	2.880	31	2.880	21	2.885	21	2.876
2.75	21	2.328	21	2.899	21	2.899	21	2.899	21	2.895	21	2.890	21	2.890	21	2.890	21	2.890	21	2.880
2.80	21	2.328	21	2.290	21	2.904	21	2.909	21	2.899	21	2.899	21	2.895	21	2.890	21	2.895	21	2.690
2.85	21	2.328	21	2.328	21	2.909	21	2.909	21	2.909	21	2.904	21	2.899	21	2.899	21	2.899	21	2.895
2.90	21	2.328	21	2.328	21	2.918	21	2.914	21	2.909	21	2.909	21	2.909	21	2.909	21	2.909	21	2.899
2.95	21	2.328	21	2.328	21	2.290	21	2.918	21	2.918	21	2.918	21	2.914	21	2.909	21	2.909	21	2.909
3.00	21	2.326	21	2.328	21	2.328	21	2.928	21	2.923	21	2.918	21	2.918	21	2.918	21	2.914	21	2.909
3.05	21	2.328	21	2.328	21	2.328	21	2.928	21	2.928	21	2.928	21	2.928	21	2.923	21	2.918	21	2.918
3.10	21	2.366	21	2.328	21	2.328	21	2.933	21	2.933	21	2.933	21	2.928	21	2.928	21	2.928	21	2.923
3.15	21	2.366	21	2.328	21	2.328	21	2.328	21	2.936	21	2.935	21	2.933	21	2.935	21	2.933	21	2.928
3.20	21	2.347	21	2.328	21	2.328	21	2.328	21	2.947	21	2.940	21	2.937	21	2.935	21	2.937	21	2.933
3.25	21	2.357	21	2.366	21	2.328	21	2.328	21	2.328	21	2.947	21	2.947	21	2.942	21	2.947	21	2.936
3.30	21	2.357	21	2.366	21	2.328	21	2.328	21	2.328	21	2.952	21	2.947	21	2.947	21	2.952	21	2.947
3.35	21	2.357	21	2.357	21	2.326	21	2.328	21	2.328	21	2.957	21	2.957	21	2.957	21	2.957	21	2.947
3.40	21	2.361	21	2.357	21	2.366	21	2.328	21	2.328	21	2.328	21	2.957	21	2.957	21	2.957	21	2.957
3.45	21	2.366	21	2.366	21	2.366	21	2.328	21	2.328	21	2.328	21	2.966	21	2.961	21	2.957	21	2.957

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TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	P*	2.05 T*	|R*|	2.10 T*	|R*|	2.15 T*	|R*|	2.20 T*	|R*|	2.25 T*	|R*|	2.30 T*	|R*|	2.35 T*	|R*|	2.40 T*	|R*|	2.45 T*	|R*|	2.50 T*	|R*|
0.00	*	0.102	0	0.098	0	0.095	0	0.092	0	0.088	0	0.085	0	0.083	0	0.080	0	0.078	0	0.076	0
0.05	1	0.365	1	0.354	1	0.342	1	0.332	1	0.320	1	0.311	1	0.456	1	0.156	1	0.156	1	0.148	1
0.10	1	0.510	1	0.496	1	0.482	1	0.469	1	0.457	1	0.445	1	0.434	1	0.422	1	0.412	1	0.400	1
0.15	1	0.629	1	0.614	1	0.602	1	0.588	1	0.576	1	0.562	1	0.550	1	0.538	1	0.526	1	0.514	1
0.20	1	0.728	1	0.714	1	0.702	1	0.690	1	0.676	1	0.662	1	0.650	1	0.638	1	0.626	1	0.614	1
0.25	1	0.805	1	0.786	1	0.786	1	0.767	1	0.767	1	0.767	1	0.748	1	0.729	1	0.729	1	0.700	1
0.30	2	1.266	2	1.247	2	1.233	2	1.214	2	1.200	2	1.181	2	1.167	2	1.147	2	1.128	2	1.114	2
0.35	2	1.352	2	1.338	2	1.319	2	1.305	2	1.290	2	1.271	2	1.257	2	1.238	2	1.224	2	1.209	2
0.40	2	1.424	2	1.409	2	1.395	2	1.381	2	1.366	2	1.352	2	1.338	2	1.319	2	1.305	2	1.290	2
0.45	2	1.490	2	1.476	2	1.462	2	1.452	2	1.433	2	1.428	2	1.409	2	1.395	2	1.381	2	1.366	2
0.50	2	1.547	2	1.538	2	1.524	2	1.509	2	1.500	2	1.485	2	1.471	2	1.457	2	1.443	2	1.433	2
0.55	2	1.604	2	1.566	2	1.566	2	1.566	2	1.566	2	1.528	2	1.528	2	1.528	2	1.509	2	1.488	2
0.60	2	1.643	2	1.643	2	1.604	2	1.604	2	1.604	2	1.566	3	1.566	3	1.566	3	1.566	3	1.566	3
0.65	3	2.157	3	2.147	3	1.643	3	2.119	3	2.109	3	2.090	3	2.081	3	2.066	3	2.052	3	2.090	3
0.70	3	2.204	3	2.195	3	2.181	3	2.166	3	2.157	3	2.147	3	2.128	3	2.119	3	2.104	3	2.142	3
0.75	3	2.252	3	2.238	3	2.223	3	2.214	3	2.204	3	2.190	3	2.176	3	2.166	3	2.157	3	2.185	3
0.80	3	2.290	3	2.281	3	2.266	3	2.257	3	2.242	3	2.233	3	2.223	3	2.214	3	2.200	3	2.233	3
0.85	3	2.323	3	2.314	3	2.304	3	2.295	3	2.281	3	2.271	3	2.261	3	2.252	3	2.242	3	2.271	3
0.90	3	2.357	3	2.347	3	2.338	3	2.327	3	2.319	3	2.309	3	2.300	3	2.290	3	2.281	3	2.304	3
0.95	3	2.390	3	2.380	3	2.371	3	2.361	3	2.352	3	2.342	3	2.333	3	2.323	3	2.314	3	2.338	3
1.00	3	2.419	3	2.414	3	2.404	3	2.395	3	2.385	3	2.376	3	2.366	3	2.357	3	2.347	3	2.366	3
1.05	3	2.447	3	2.438	3	2.433	3	2.423	3	2.414	3	2.404	3	2.395	3	2.385	3	2.376	3	2.400	3
1.10	3	2.471	3	2.461	3	2.457	3	2.452	3	2.442	3	2.433	3	2.423	3	2.423	3	2.404	3	2.442	3
1.15	3	2.500	3	2.490	3	2.478	3	2.471	3	2.466	3	2.480	3	2.480	3	2.442	3	2.442	3	2.480	3
1.20	3	2.519	3	2.509	3	2.504	3	2.500	3	2.519	3	2.480	3	2.480	3	2.480	3	2.480	3	2.480	3
1.25	3	2.538	3	2.533	3	2.557	3	2.519	3	2.519	3	2.519	3	2.519	3	2.480	3	2.480	3	2.480	3
1.30	3	2.552	3	2.552	3	2.547	3	2.557	3	2.557	3	2.557	3	2.557	3	2.480	3	2.480	3	2.519	3
1.35	3	2.561	3	2.561	3	2.595	3	2.557	3	2.557	3	2.557	3	2.557	3	2.557	3	2.557	3	2.557	3
1.40	3	2.580	3	2.595	3	2.595	3	2.595	3	2.557	3	2.558	3	2.595	3	2.557	3	2.557	3	2.557	3
1.45	3	2.599	3	2.595	3	2.633	3	2.595	3	2.633	3	2.639	3	2.633	3	2.557	3	2.557	3	2.557	3
1.50	3	2.630	3	2.633	3	2.633	3	2.633	3	2.633	3	2.639	3	2.633	3	2.557	3	2.557	3	2.557	3
1.55	3	2.630	3	2.633	3	2.633	3	2.633	3	2.533	3	2.639	3	2.633	3	2.595	3	3.137	3	3.128	3
1.60	3	2.671	3	2.633	3	2.633	3	2.633	3	2.633	3	2.639	3	2.633	3	2.595	3	2.595	3	3.147	3
1.65	3	2.671	3	2.671	3	2.633	3	2.633	3	2.633	3	2.639	3	2.633	3	2.595	3	2.595	3	3.147	3
1.70	3	2.709	3	2.671	3	2.633	3	2.633	3	2.633	3	2.639	3	—	—	2.633	3	3.176	3	3.166	3

TABLE 6.20 (Continued)

GAMMA STAR	LAMBDA																			
	2.05		2.10		2.15		2.20		2.25		2.30		2.35		2.40		2.45		2.50	
	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*
1.75	31	2.709	31	2.709	31	2.709	31	2.709	31	2.671	31	2.633	31	2.633	31	2.633	31	3.130	31	3.185
1.80	31	2.709	31	2.709	31	2.709	31	2.709	31	2.709	31	2.671	31	3.216	31	3.214	31	2.633	31	3.204
1.85	31	2.747	31	2.709	31	2.709	31	2.709	31	2.709	31	2.671	31	2.671	31	3.233	31	3.223	31	3.218
1.90	31	2.747	31	2.709	31	2.709	31	2.709	31	2.709	31	2.709	31	2.671	31	3.242	31	3.242	31	3.233
1.95	31	2.747	31	2.709	31	2.709	31	2.709	31	2.709	31	2.709	31	2.709	31	3.271	31	3.252	31	3.252
2.00	31	2.766	31	2.747	31	2.709	31	2.709	31	2.709	31	2.708	31	2.709	31	2.708	31	3.271	31	3.261
2.05	31	2.785	31	2.785	31	2.785	31	2.747	31	2.709	31	2.709	31	2.709	31	2.709	31	3.280	31	3.280
2.10	31	2.785	31	2.785	31	2.785	31	2.785	31	2.785	31	2.747	31	3.304	31	3.309	31	3.295	31	3.290
2.15	31	2.780	31	2.785	31	2.785	31	2.785	31	2.785	31	2.747	31	2.318	31	3.328	31	3.309	31	3.299
2.20	31	2.823	31	2.785	31	2.785	31	2.785	31	2.785	31	2.747	31	2.747	31	3.337	31	3.318	31	3.318
2.25	31	2.814	31	2.785	31	2.785	31	2.785	31	2.785	31	2.785	31	2.747	31	3.347	31	3.328	31	3.328
2.30	31	2.818	31	2.823	31	2.823	31	2.785	31	2.785	31	2.785	31	2.785	31	3.356	31	3.342	31	3.337
2.35	31	2.828	31	2.823	31	2.861	31	2.785	31	2.785	31	2.785	31	2.785	31	2.785	31	3.356	31	3.347
2.40	31	2.833	31	2.861	31	2.861	31	2.823	31	2.861	31	2.823	31	2.785	31	2.785	31	3.366	31	3.361
2.45	31	2.842	31	2.861	31	2.857	31	2.861	31	2.861	31	2.861	31	3.395	31	3.395	31	3.375	31	3.371
2.50	31	2.852	31	2.852	31	2.899	31	2.861	31	2.861	31	2.861	31	2.823	31	3.404	31	2.785	31	3.380
2.55	31	2.861	31	2.657	31	2.899	31	2.861	31	2.861	31	2.861	31	2.823	31	2.823	31	3.395	31	3.390
2.60	31	2.866	31	2.864	31	2.880	31	2.899	31	2.861	31	2.861	31	2.861	31	2.823	31	3.404	31	3.399
2.65	31	2.871	31	2.671	31	2.890	31	2.904	31	2.861	31	2.861	31	2.861	31	2.861	31	3.414	31	3.409
2.70	31	2.880	31	2.876	31	2.890	31	2.899	31	2.861	31	2.861	31	2.861	31	2.823	31	3.423	31	3.418
2.75	31	2.885	31	2.880	31	2.899	31	2.899	31	2.861	31	2.861	31	2.899	31	2.861	31	2.823	31	3.423
2.80	31	2.890	31	2.890	31	2.880	31	2.899	31	2.861	31	2.861	31	2.899	31	3.437	31	3.437	31	3.442
2.85	31	2.890	31	2.890	31	2.890	31	2.899	31	2.399	31	2.861	31	2.861	31	2.861	31	2.861	31	3.442
2.90	31	2.899	31	2.899	31	2.899	31	2.904	31	2.899	31	2.861	31	2.861	31	2.861	31	2.861	31	3.452
2.95	31	2.904	31	2.909	31	2.909	31	2.909	31	2.899	31	2.899	31	2.861	31	2.861	31	2.861	31	2.861
3.00	31	2.909	31	2.909	31	2.909	31	2.908	31	2.899	31	2.899	31	2.899	31	2.861	31	2.861	31	2.861
3.05	31	2.918	31	2.914	31	2.918	31	2.914	31	2.938	31	2.938	31	2.899	31	2.861	31	2.861	31	2.861
3.10	31	2.928	31	2.918	31	2.923	31	2.918	31	2.918	31	2.938	31	2.938	31	2.861	31	2.861	31	2.861
3.15	31	2.933	31	2.928	31	2.928	31	2.928	31	2.923	31	2.938	31	2.938	31	2.938	31	2.899	31	2.480
3.20	31	2.935	31	2.933	31	2.933	31	2.928	31	2.928	31	2.938	31	2.938	31	2.938	31	2.938	31	2.899
3.25	31	2.942	31	2.937	31	2.935	31	2.933	31	2.933	31	2.939	31	2.938	31	2.938	31	2.938	31	2.899
3.30	31	2.947	31	2.947	31	2.942	31	2.937	31	2.936	31	2.935	31	2.933	31	2.938	31	2.938	31	2.938
3.35	31	2.952	31	2.947	31	2.947	31	2.947	31	2.947	31	2.948	31	2.937	31	2.938	31	2.938	31	2.938
3.40	31	2.957	31	2.957	31	2.952	31	2.947	31	2.947	31	2.847	31	2.976	31	2.976	31	2.938	31	2.938

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TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	2.55 |R*|	2.55 T*	2.60 |R*|	2.60 T*	2.65 |R*|	2.65 T*	2.70 |R*|	2.70 T*	2.75 |R*|	2.75 T*	2.80 |R*|	2.80 T*	2.85 |R*|	2.85 T*	2.90 |R*|	2.90 T*	2.95 |R*|	2.95 T*	3.00 |R*|	3.00 T*
0.00	0	0.073	0	0.071	0	0.069	0	0.067	0	0.065	0	0.068	0	0.062	0	0.061	0	0.059	0	0.058
0.05	0	0.141	0	0.141	0	0.141	0	0.134	0	0.130	0	0.123	0	0.124	0	0.121	0	0.118	0	0.115
0.10	1	0.391	1	0.381	1	0.371	1	0.363	1	0.354	1	0.346	1	0.336	1	0.328	1	0.321	1	0.314
0.15	1	0.502	1	0.492	1	0.480	1	0.469	1	0.459	1	0.449	1	0.439	1	0.430	1	0.420	1	0.412
0.20	1	0.600	1	0.590	1	0.579	1	0.567	1	0.555	1	0.545	1	0.533	1	0.524	1	0.512	1	0.502
0.25	1	0.690	1	0.675	1	0.667	1	0.652	1	0.643	1	0.631	1	0.619	1	0.609	1	0.598	1	0.588
0.30	1	0.788	1	0.748	1	0.729	1	0.729	1	0.729	1	0.709	1	0.709	1	0.686	1	0.690	1	0.664
0.35	2	1.190	2	1.176	2	1.162	2	1.143	2	1.128	2	1.118	2	1.095	2	1.081	1	0.729	1	0.729
0.40	2	1.276	2	1.262	2	1.247	2	1.228	2	1.214	2	1.200	2	1.186	2	1.171	2	1.152	2	1.138
0.45	2	1.352	2	1.338	2	1.324	2	1.309	2	1.290	2	1.276	2	1.262	2	1.247	2	1.233	2	1.219
0.50	2	1.419	2	1.405	2	1.390	2	1.376	2	1.362	2	1.347	2	1.333	2	1.319	2	1.309	2	1.295
0.55	2	1.490	2	1.466	2	1.452	2	1.438	2	1.424	2	1.412	2	1.400	2	1.385	2	1.371	2	1.357
0.60	2	1.528	2	1.490	2	1.490	2	1.490	2	1.490	2	1.490	2	1.452	2	1.452	2	1.452	2	1.433
0.65	3	2.023	3	1.528	3	1.522	3	1.528	3	1.528	3	1.528	3	1.490	3	1.490	3	1.490	3	1.490
0.70	3	2.081	3	2.009	3	2.052	3	2.038	3	2.023	3	2.014	3	1.995	3	1.985	3	1.966	3	1.957
0.75	3	2.176	3	2.066	3	2.104	3	2.090	3	2.081	3	2.066	3	2.052	3	2.039	3	2.022	3	2.022
0.80	3	2.219	3	2.116	3	2.152	3	2.138	3	2.128	3	2.114	3	2.100	3	2.090	3	2.081	3	2.062
0.85	3	2.257	3	2.166	3	2.195	3	2.185	3	2.176	3	2.162	3	2.147	3	2.139	3	2.128	3	2.114
0.90	3	2.295	3	2.204	3	2.238	3	2.223	3	2.214	3	2.204	3	2.195	3	2.181	3	2.166	3	2.157
0.95	3	2.326	3	2.247	3	2.271	3	2.261	3	2.252	3	2.242	3	2.233	3	2.224	3	2.214	3	2.200
1.00	3	2.364	3	2.285	3	2.309	3	2.300	3	2.290	3	2.281	3	2.271	3	2.261	3	2.252	3	2.238
1.05	3	2.404	3	2.319	3	2.342	3	2.333	3	2.323	3	2.314	3	2.328	3	2.295	3	2.285	3	2.276
1.10	3	2.442	3	2.352	3	2.366	3	2.366	3	2.366	3	2.366	3	2.366	3	2.328	3	2.328	3	2.328
1.15	3	2.442	3	2.404	3	2.404	3	2.404	3	2.404	3	2.404	3	2.366	3	2.328	3	2.328	3	2.366
1.20	3	2.480	3	2.404	3	2.404	3	2.404	3	2.404	3	2.404	3	2.404	3	2.404	3	2.404	3	2.404
1.25	3	2.480	3	2.480	3	2.480	3	2.480	3	2.480	3	2.442	3	2.957	3	2.404	3	2.404	3	2.404
1.30	3	2.480	3	2.480	3	2.480	3	2.480	3	2.480	3	2.404	3	2.442	3	2.976	3	2.966	3	2.957
1.35	3	2.519	3	2.480	3	2.480	3	2.480	3	2.404	3	2.404	3	3.033	3	2.999	3	2.995	3	2.985
1.40	3	2.519	3	2.480	3	2.480	3	2.442	3	2.442	3	2.442	3	3.057	3	3.023	3	3.014	3	3.009
1.45	3	2.557	3	2.519	3	2.480	3	3.057	3	2.480	3	3.018	3	3.080	3	3.052	3	3.042	3	3.033
1.50	3	2.557	3	2.519	3	3.042	3	3.080	3	3.052	3	3.042	3	3.108	3	3.071	3	3.061	3	3.061
1.55	3	3.142	3	3.095	3	3.061	3	3.099	3	3.071	3	3.061	3	3.080	3	3.096	3	3.090	3	3.080
1.60	4	3.161	4	3.118	4	3.090	4	3.118	4	3.095	4	3.090	4	3.092	4	3.118	4	3.090	4	3.080
1.65	4	3.142	4	3.137	4	3.109	4	3.137	4	3.118	4	3.109	4	3.118	4	3.118	4	3.109	4	3.098
1.70	4	3.161	4	3.156	4	3.147	4	3.137	4	3.137	4	3.128	4	3.118	4	3.118	4	3.109	4	3.098

TABLE 6.20 (Continued)

GAMMA STAR	LAMBDA 2.55		2.60		2.65		2.70		2.75		2.80		2.85		2.90		2.95		3.00	
	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|	T*	|R*|
1.75	3.176	4	3.176	4	3.166	4	3.156	4	3.156	4	3.147	4	3.137	4	3.137	4	3.128	4	3.118	4
1.80	3.195	4	3.190	4	3.185	4	3.185	4	3.176	4	3.166	4	3.166	4	3.156	4	3.147	4	3.137	4
1.85	3.214	4	3.204	4	3.204	4	3.195	4	3.190	4	3.185	4	3.176	4	3.171	4	3.165	4	3.156	4
1.90	3.226	4	3.223	4	3.214	4	3.214	4	3.204	4	3.204	4	3.195	4	3.185	4	3.185	4	3.176	4
1.95	3.240	4	3.237	4	3.233	4	3.228	4	3.223	4	3.214	4	3.214	4	3.204	4	3.199	4	3.195	4
2.00	3.261	4	3.252	4	3.247	4	3.240	4	3.233	4	3.233	4	3.223	4	3.223	4	3.214	4	3.214	4
2.05	3.271	4	3.271	4	3.261	4	3.256	4	3.252	4	3.247	4	3.240	4	3.233	4	3.233	4	3.223	4
2.10	3.285	4	3.280	4	3.276	4	3.271	4	3.266	4	3.261	4	3.256	4	3.252	4	3.242	4	3.237	4
2.15	3.299	4	3.290	4	3.290	4	3.285	4	3.280	4	3.271	4	3.271	4	3.261	4	3.261	4	3.252	4
2.20	3.309	4	3.309	4	3.299	4	3.299	4	3.290	4	3.290	4	3.280	4	3.280	4	3.271	4	3.271	4
2.25	3.323	4	3.318	4	3.318	4	3.309	4	3.309	4	3.299	4	3.299	4	3.290	4	3.299	4	3.280	4
2.30	3.337	4	3.328	4	3.328	4	3.318	4	3.318	4	3.309	4	3.309	4	3.304	4	3.299	4	3.295	4
2.35	3.347	4	3.342	4	3.337	4	3.333	4	3.328	4	3.328	4	3.318	4	3.318	4	3.309	4	3.309	4
2.40	3.356	4	3.356	4	3.347	4	3.347	4	3.337	4	3.337	4	3.333	4	3.328	4	3.323	4	3.318	4
2.45	3.375	4	3.366	4	3.356	4	3.356	4	3.352	4	3.347	4	3.347	4	3.337	4	3.337	4	3.328	4
2.50	3.375	4	3.375	4	3.366	4	3.366	4	3.361	4	3.356	4	3.356	4	3.347	4	3.356	4	3.342	4
2.55	3.385	4	3.385	4	3.380	4	3.375	4	3.375	4	3.366	4	3.366	4	3.361	4	3.356	4	3.356	4
2.60	3.395	4	3.395	4	3.390	4	3.385	4	3.385	4	3.375	4	3.375	4	3.371	4	3.366	4	3.366	4
2.65	3.404	4	3.404	4	3.399	4	3.395	4	3.395	4	3.385	4	3.385	4	3.386	4	3.375	4	3.375	4
2.70	3.414	4	3.414	4	3.404	3	3.404	4	3.404	4	3.395	4	3.395	4	3.390	4	3.385	4	3.385	4
2.75	3.423	4	3.423	4	3.414	4	3.414	4	3.418	4	3.414	4	3.404	4	3.399	4	3.395	4	3.390	4
2.80	3.433	4	3.428	4	3.423	4	3.423	4	3.423	4	3.423	4	3.414	4	3.408	4	3.404	4	3.404	4
2.85	3.442	4	3.433	4	3.433	4	3.433	4	3.433	4	3.423	4	3.423	4	3.414	4	3.414	4	3.414	4
2.90	3.447	4	3.442	4	3.442	4	3.447	4	3.442	4	3.439	4	3.433	4	3.423	4	3.423	4	3.423	4
2.95	3.452	4	3.452	4	3.452	4	3.452	4	3.452	4	3.442	4	3.442	4	3.433	4	3.433	4	3.428	4
3.00	3.461	3	3.461	3	3.461	3	3.461	3	3.461	3	3.452	4	3.452	4	3.442	4	3.442	4	3.433	4
3.05	3.461	3	3.471	3	3.471	3	3.471	3	3.466	3	3.461	4	3.461	4	3.452	4	3.447	4	3.442	4
3.10	3.480	4	3.471	4	3.480	4	3.475	4	3.471	4	3.471	4	3.471	4	3.461	3	3.452	3	3.452	3
3.15	3.490	3	3.480	3	3.485	3	3.480	3	3.480	3	3.480	3	3.480	3	3.471	3	3.471	3	3.461	3
3.20	3.490	3	3.490	3	3.490	3	3.490	3	3.490	3	3.480	3	3.490	3	3.480	3	3.475	3	3.471	3
3.25	3.499	3	3.495	3	3.499	3	3.499	3	3.490	3	3.490	3	3.495	9	3.485	3	3.480	3	3.480	3
3.30	2.932	31	2.899	31	3.504	31	3.499	31	3.499	31	3.499	31	3.499	31	3.490	31	3.490	31	3.490	31
3.35	2.899	31	2.899	31	3.509	31	3.509	31	3.509	31	3.508	31	3.499	31	3.499	31	3.499	31	3.490	31
3.40	2.938	31	2.899	31	3.509	31	3.509	31	3.509	31	3.508	31	3.509	31	3.499	31	3.499	31	3.490	31
3.45	2.938	31	2.938	31	2.899	31	3.518	31	3.509	31	3.508	31	3.509	31	3.508	31	3.499	31	3.499	31

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TABLE 6.20 (Continued)

TABLE 6.20 (Continued)

GAMMA STAR	LAMBDA																			
	3.05		3.10		3.15		3.20		3.25		3.30		3.35		3.40		3.45		3.50	
	Re	Te	Re	Te	Re	Te	Re	Te	Re	Te	Re	Te	Re	Te	Re	Te	Re	Te	Re	Te
1.75	4	3.114	4	3.109	4	3.099	4	3.095	4	3.090	4	3.080	4	3.071	4	3.066	4	3.061	4	3.052
1.80	4	3.137	4	3.128	4	3.181	4	3.114	4	3.109	4	3.099	4	3.095	4	3.085	4	3.080	4	3.071
1.85	4	3.156	4	3.147	4	3.137	4	3.137	4	3.128	4	3.118	4	3.114	4	3.109	4	3.099	4	3.090
1.90	4	3.171	4	3.166	4	3.156	4	3.152	4	3.147	4	3.137	4	3.133	4	3.128	4	3.118	4	3.114
1.95	4	3.185	4	3.185	4	3.176	4	3.171	4	3.166	4	3.156	4	3.152	4	3.147	4	3.137	4	3.133
2.00	4	3.204	4	3.199	4	3.195	4	3.185	4	3.185	4	3.176	4	3.166	4	3.166	4	3.156	4	3.152
2.05	4	3.223	4	3.214	4	3.209	4	3.204	4	3.195	4	3.195	4	3.185	4	3.180	4	3.176	4	3.166
2.10	4	3.233	4	3.233	4	3.223	4	3.218	4	3.214	4	3.209	4	3.204	4	3.195	4	3.195	4	3.204
2.15	4	3.252	4	3.241	4	3.237	4	3.233	4	3.233	4	3.218	4	3.218	4	3.242	4	3.242	4	3.204
2.20	4	3.261	4	3.261	4	3.252	4	3.252	4	3.240	4	3.237	4	3.233	4	3.240	4	3.242	4	3.242
2.25	4	3.280	4	3.271	4	3.271	4	3.261	4	3.261	4	3.280	4	3.280	4	3.280	4	3.242	4	3.242
2.30	4	3.290	4	3.285	4	3.280	4	3.276	4	3.271	4	3.266	4	3.280	4	3.280	4	3.280	4	3.242
2.35	4	3.295	4	3.299	4	3.295	4	3.290	4	3.318	4	3.280	4	3.280	4	3.318	4	3.280	4	3.280
2.40	4	3.318	4	3.309	4	3.318	4	3.299	4	3.309	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318
2.45	4	3.328	4	3.323	4	3.318	4	3.314	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318
2.50	4	3.337	4	3.337	4	3.328	4	3.356	4	3.318	4	3.314	4	3.318	4	3.318	4	3.318	4	3.318
2.55	4	3.347	4	3.347	4	3.337	4	3.337	4	3.356	4	3.356	4	3.318	4	3.356	4	3.318	4	3.318
2.60	4	3.356	4	3.356	4	3.352	4	3.347	4	3.356	4	3.356	4	3.356	4	3.395	4	3.318	4	3.318
2.65	4	3.366	4	3.366	4	3.361	4	3.356	4	3.356	4	3.356	4	3.356	4	3.395	4	3.318	4	3.318
2.70	4	3.375	4	3.375	4	3.375	4	3.395	4	3.395	4	3.395	4	3.356	4	3.356	4	3.318	4	3.318
2.75	4	3.385	4	3.385	4	3.385	4	3.375	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395
2.80	4	3.394	4	3.392	4	3.390	4	3.385	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395
2.85	4	3.404	4	3.404	4	3.404	4	3.393	4	3.390	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395
2.90	4	3.414	4	3.414	4	3.409	4	3.433	4	3.433	4	3.433	4	3.395	4	3.395	4	3.395	4	3.395
2.95	4	3.423	4	3.423	4	3.418	4	3.414	4	3.433	4	3.433	4	3.395	4	3.395	4	3.395	4	3.395
3.00	4	3.433	4	3.433	4	3.423	4	3.423	4	3.423	4	3.433	4	3.433	4	3.395	4	3.395	4	3.395
3.05	4	3.442	4	3.437	4	3.433	4	3.433	4	3.428	4	3.435	4	3.435	4	3.433	4	3.395	4	3.395
3.10	4	3.452	4	3.442	4	3.442	4	3.442	4	3.471	4	3.471	4	3.433	4	3.433	4	3.433	4	3.395
3.15	4	3.456	4	3.452	4	3.452	4	3.447	4	3.471	4	3.471	4	3.471	4	3.471	4	3.395	4	3.395
3.20	4	3.461	4	3.461	4	3.461	4	3.452	4	3.452	4	3.471	4	3.471	4	3.471	4	3.433	4	3.395
3.25	4	3.471	4	3.471	4	3.461	4	3.461	4	3.461	4	3.471	4	3.471	4	3.471	4	3.471	4	3.471
3.30	4	3.480	4	3.471	4	3.471	4	3.471	4	3.466	4	3.461	4	3.471	4	3.271	4	3.471	4	3.471
3.35	4	3.480	4	3.480	4	3.480	4	3.475	4	3.471	4	3.471	4	3.066	4	3.474	4	3.471	4	3.471
3.40	4	3.490	4	3.490	4	3.485	4	3.480	4	3.480	4	3.509	4	3.509	4	3.474	4	3.471	4	3.471
3.45	4	3.499	4	3.495	4	3.490	4	3.490	4	3.490	4	3.485	4	3.509	4	3.509	4	3.071	4	3.471

TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	3.55		3.60		3.65		3.70		3.75		3.80		3.85		3.90		3.95		4.00	
	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*	R*	T*
0.00	*	0.046	*	0.045	*	0.044	*	0.043	*	0.042	*	0.042	*	0.041	*	0.040	*	0.040	*	0.039
0.05	0	0.091	0	0.090	0	0.088	0	0.086	0	0.085	0	0.083	0	0.082	0	0.081	0	0.079	0	0.078
0.10	0	0.133	0	0.133	0	0.133	0	0.133	0	0.129	0	0.125	0	0.123	0	0.121	0	0.119	0	0.117
0.15	1	0.330	1	0.323	1	0.317	1	0.312	1	0.307	1	0.301	1	0.295	1	0.290	1	0.285	1	0.280
0.20	1	0.408	1	0.400	1	0.395	1	0.387	1	0.381	1	0.373	1	0.367	1	0.361	1	0.355	1	0.350
0.25	1	0.484	1	0.477	1	0.469	1	0.459	1	0.453	1	0.415	1	0.438	1	0.430	1	0.424	1	0.416
0.30	1	0.557	1	0.548	1	0.538	1	0.531	1	0.521	1	0.514	1	0.505	1	0.498	1	0.490	1	0.482
0.35	1	0.624	1	0.614	1	0.605	1	0.598	1	0.588	1	0.579	1	0.571	1	0.562	1	0.555	1	0.545
0.40	1	0.690	1	0.690	1	0.671	1	0.671	1	0.650	1	0.648	1	0.690	1	0.624	1	0.614	1	0.607
0.45	1	1.062	2	1.050	2	1.036	2	1.024	2	1.009	2	0.690	2	0.690	2	0.690	2	0.671	2	0.652
0.50	2	1.143	2	1.128	2	1.114	2	1.100	2	1.086	2	1.076	2	1.062	2	1.047	2	1.033	2	1.024
0.55	2	1.214	2	1.206	2	1.186	2	1.171	2	1.162	2	1.147	2	1.133	2	1.118	2	1.107	2	1.095
0.60	2	1.281	2	1.266	2	1.252	2	1.238	2	1.228	2	1.214	2	1.200	2	1.190	2	1.176	2	1.162
0.65	2	1.337	2	1.328	2	1.314	2	1.300	2	1.290	2	1.276	2	1.266	2	1.252	2	1.238	2	1.228
0.70	2	1.414	2	1.395	2	1.376	2	1.376	2	1.347	2	1.333	2	1.324	2	1.309	2	1.300	2	1.286
0.75	2	1.433	2	1.414	2	1.414	2	1.414	2	1.414	2	1.376	2	1.376	2	1.376	2	1.376	2	1.357
0.80	3	1.919	3	1.904	3	1.890	3	1.876	3	1.862	3	1.418	3	1.414	3	1.416	3	1.414	3	1.414
0.85	3	1.971	3	1.957	3	1.947	3	1.933	3	1.919	3	1.904	3	1.890	3	1.881	3	1.862	3	1.852
0.90	3	2.021	3	2.014	3	2.000	3	1.985	3	1.976	3	1.957	3	1.947	3	1.933	3	1.919	3	1.909
0.95	3	2.071	3	2.062	3	2.047	3	2.033	3	2.021	3	2.014	3	2.000	3	1.985	3	1.976	3	1.962
1.00	3	2.119	3	2.104	3	2.095	3	2.081	3	2.071	3	2.062	3	2.047	3	2.033	3	2.021	3	2.009
1.05	3	2.157	3	2.147	3	2.138	3	2.128	3	2.114	3	2.100	3	2.090	3	2.081	3	2.071	3	2.057
1.10	3	2.214	3	2.250	3	2.175	3	2.166	3	2.157	3	2.147	3	2.133	3	2.123	3	2.109	3	2.100
1.15	3	2.252	3	2.252	3	2.214	3	2.214	3	2.214	3	2.214	3	2.171	3	2.176	3	2.176	3	2.138
1.20	3	2.252	3	2.252	3	2.252	3	2.252	3	2.252	3	2.214	3	2.214	3	2.176	3	2.176	3	2.176
1.25	3	2.290	3	2.290	3	2.290	3	2.290	3	2.252	3	2.252	3	2.252	3	2.252	3	2.252	3	2.214
1.30	4	2.818	4	2.842	3	2.833	4	2.823	3	2.776	3	2.808	3	2.795	3	2.780	3	2.766	3	2.252
1.35	4	2.852	4	2.876	3	2.861	4	2.852	3	2.814	3	2.833	3	2.823	3	2.818	3	2.804	3	2.795
1.40	4	2.885	4	2.899	3	2.899	4	2.890	3	2.842	3	2.871	3	2.861	3	2.852	3	2.838	3	2.828
1.45	4	2.914	4	2.933	3	2.928	4	2.918	4	2.880	3	2.899	3	2.890	3	2.880	3	2.871	3	2.861
1.50	4	2.542	4	2.961	3	2.957	4	2.947	4	2.933	3	2.928	3	2.918	3	2.909	4	2.899	3	2.890
1.55	4	2.971	4	2.985	3	2.976	4	2.971	4	2.966	3	2.957	4	2.947	4	2.935	4	2.928	4	2.918
1.60	4	2.995	4	3.014	4	3.004	4	2.995	4	2.985	3	2.980	4	2.976	4	2.966	4	2.957	4	2.947
1.65	4	3.023																		

TABLE 6.20 (Continued)

LAMBDA

GAMMA STAR	3.55 T*	|P*|	3.60 T*	|R*|	3.65 T*	|R*|	3.70 T*	|P*|	3.75 T*	|R*|	3.80 T*	|R*|	3.85 T*	|R*|	3.90 T*	|R*|	3.95 T*	|R*|	4.00 T*	|R*|
1.75	3.042	4	3.033	4	3.028	4	3.023	4	3.014	4	3.008	4	2.995	4	2.996	4	2.980	4	2.976	4
1.80	3.066	4	3.061	4	3.052	4	3.042	4	3.033	4	3.028	4	3.023	4	3.018	4	3.004	4	2.995	4
1.85	3.085	4	3.080	4	3.071	4	3.061	4	3.061	4	3.052	4	3.042	4	3.033	4	3.033	4	3.023	4
1.90	3.109	4	3.099	4	3.090	4	3.085	4	3.080	4	3.071	4	3.066	4	3.090	4	3.052	4	3.042	4
1.95	3.128	4	3.118	4	3.114	4	3.109	4	3.128	4	3.090	4	3.085	4	3.080	4	3.090	4	3.090	4
2.00	3.147	4	3.166	4	3.166	4	3.128	4	3.118	4	3.128	4	3.128	4	3.128	4	3.089	4	3.090	4
2.05	3.161	4	3.156	4	3.166	4	3.166	4	3.166	4	3.166	4	3.128	4	3.128	4	3.128	4	3.128	4
2.10	3.204	4	3.204	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4
2.15	3.204	4	3.204	4	3.204	4	3.204	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4	3.156	4
2.20	3.242	4	3.242	4	3.242	4	3.242	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4
2.25	3.242	4	3.242	4	3.242	4	3.242	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4	3.166	4
2.30	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.204	4
2.35	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.204	4
2.40	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4
2.45	3.316	4	3.318	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4
2.50	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4
2.55	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.242	4	3.242	4	3.242	4	3.242	4	3.242	4
2.60	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.280	4	3.280	4	3.833	4	3.242	4	3.242	4
2.65	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.280	4	3.280	4	3.837	4	3.833	4
2.70	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.280	4	3.280	4	3.852	4	3.842	4
2.75	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.871	4	3.280	4	3.861	4
2.80	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.318	4	3.860	4	3.871	4
2.85	3.395	4	3.395	4	3.358	4	3.358	4	3.318	4	3.918	4	3.318	4	3.318	4	3.890	4	3.880	4
2.90	3.395	4	3.395	4	3.395	4	3.395	4	3.356	4	3.928	4	3.318	4	3.918	4	3.318	4	3.899	4
2.95	3.395	4	3.395	4	3.395	4	3.395	4	3.356	4	3.356	4	3.928	4	3.926	4	3.923	4	3.918	4
3.00	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.356	4	3.942	4	3.937	4	3.937	4	3.928	4
3.05	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.356	4	3.947	4	3.947	4	3.937	4
3.10	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.966	4	3.956	4	3.956	4	3.947	4
3.15	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.966	4	3.966	4	3.961	4
3.20	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.395	4	3.975	4	3.975	4	3.971	4
3.25	3.395	4	3.395	4	3.395	4	3.433	4	3.395	4	3.395	4	3.395	4	3.395	4	3.985	4	3.980	4
3.30	3.471	4	3.471	4	3.471	4	3.471	4	3.433	4	3.395	4	3.395	4	3.395	4	3.395	4	3.990	4
3.35	3.471	4	3.471	4	3.471	4	3.471	4	3.433	4	4.008	4	4.008	4	4.004	4	3.395	4	3.994	4
3.40	3.471	4	3.471	4	3.471	4	3.471	4	3.471	4	4.013	4	4.009	4	4.009	4	4.004	4	3.990	4
3.45	3.471	4	3.471	4	3.471	4	3.471	4	3.471	4	3.433	4	4.013	4	4.013	4	4.013	4	4.004	4

APPENDIX II

The following are sample contractual requirements for the application of Bayesian Reliability Tests to the purchase of commercial equipment. Blanks represent information to be supplied by the user. Bold face entries are not part of the sample but refer to notes to the user listed after the sample. These must be considered by the user in implementing the sample paragraphs. The sample paragraphs must be placed in appropriate sections of the statement of work and/or specification and backed up by appropriate data items in the contract data requirements list. On the latter, it is believed that only standard data items (the same ones used for classical reliability demonstration requirements) are needed for the Bayesian approach.

x. Reliability. The _____ shall be designed to achieve a mean time between failure (MTBF) of no less than _____ hours. This figure is designated the upper test MTBF (θ_0) for demonstration purposes. The lower test MTBF (θ_1) is defined as _____ hours. **(SEE NOTE 1)**

y. Reliability Demonstration. The Reliability of the _____ shall be demonstrated in accordance with MIL-STD-781, except that a Bayesian Reliability Test may be used in lieu of the test plans listed in MIL-STD-781. The test to be used shall be determined as follows:

y.1. The contractor shall provide all available data showing operating time, number of failures and use environment by individual item of equipment identical to the unit purchased. The procuring activity will determine what portion, if any, of the data may be used to establish a prior for the reliability test and may supply additional data, if available. **(See Note 2).**

y.2. Data approved by the procuring activity will be tested for adequacy by the criteria of formula 4.5 in RADC-TR -_____. **(See**

Note 3). If no approved data is available, or if the data fails the adequacy criteria, a Bayesian test will not be used and test plan ____ of MIL-STD-781 will be used instead.

y.3. If adequate approved data is available, formulas 4.2 - 4.4 of RADC-TR-_____ **(See Note 3)** will be used to determine the prior parameters (λ) and (γ). The test parameter (γ^*) will be computed by the formula: $\gamma^* = \gamma/\theta_0$.

y.4. The values of (γ^*) and (λ) will be used with table ____ of RADC-TR-78-241 **(See Note 4)** to determine the total test time (T^*) in multiples of θ_0 and the allowable value of failure (r^*). In using the table, the following procedures will be followed:

y.4.1. If the value of (γ^*), (λ) or both fall outside the limits of the table, formulas 5.7 and 5.8 of RADC-TR-_____ **(SEE NOTE 3)** will be used to compute values within the table limits, which shall be used in lieu of the original values.

y.4.2. A table entry of (*****) shall negate the use of a Bayesian test, and test plan _____ of MIL-STD-781 shall be used instead **(See Note 5)**.

y.4.3. A table entry of (0,0.0) for (r^*,T^*) shall not result in acceptance without testing. Instead the unit will be tested for _____ hours with _____ failures allowed **(See Note 6)**.

y.5. In the event the test is not successful, the contractor shall propose appropriate corrective action in accordance with para 5.9 of MIL-STD-781C. A retest will be performed using test plan ____ of MIL-STD-781 **(See Note 7)**.

y.6. These test procedures, the test environment and monitoring provisions will be submitted for approval by the procuring activity and modified as necessary before the test begins **(See Note 8)**.

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NOTES TO USERS

1. The ratio of θ_0/θ_1 specified in para x, must match the tests specified in para y.3 and y.4. It must also match the default test in y.2.

2. It is essential that the data used for the prior be realistic (i.e., be for the same equipment under the desired use environment) and complete (i.e., include information on poor units as well as good units).

3. Insert the TR number assigned to this report (shown on the front cover).

4. The appropriate table must be selected to match the desired discrimination ratio (θ_0/θ_1), the desired types of consumers and producers risks, and the desired value of the risks. If $\theta_0/\theta_1 = 2$, and average risks of 10% are desired, reference may be made to Appendix I of this report instead of to RADC-TR-78-241, thus eliminating one reference document.

5. The same MIL-STD-781 test plan referenced in para y.2 should be referenced here.

6. This is the minimum test and it is your option to select one that will satisfy you that you haven't got a lemon. Suggestions might be to test for θ_1 hours and allow one failure (if you have a lot of confidence in the unit) or test for θ_0 hours and allow one failure (a more conservative approach, but still more generous than a classical test).

7. If the test is failed, forget the Bayesian approach and revert to the classical test. Use the same test referenced in y.2. Note that data item DI-R-7038 (Plan, Corrective Action) must be listed in the contract data requirements list. You may want some others also, (See para 30, Appendix F, MIL-STD-781C).

8. Data Item DI-R-7033 (Plan, Reliability Test) must be listed in the contract data requirements list (CDRL). Data Item DI-R-7035 (Procedures, Reliability

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Tests) will also be necessary, but may be combined with DI-R-7033 (See para 5.1.4, MIL-STD-781C). DI-R-7034 (Reports Reliability Test and Demonstration) is also needed.

APPENDIX III

The following are sample contractual provisions for the use of Bayesian Tests for production reliability control. As in Appendix II, blanks represent information to be supplied by the user and bold face entries are not part of the sample, but refer to notes to the users listed afterwards. The sample paragraphs must be placed in appropriate contractual documents and backed up by appropriate data items in the contract data requirements list. It is believed that the same data items now used for production reliability testing will suffice for the Bayesian approach. One final note: In applying this sample, it is assumed that a classical reliability demonstration has been required as a reliability qualification test and that θ_0 and θ_1 were defined for that test.

    x. Reliability acceptance testing will be performed as follows:

    x.1 Production samples:_____ (See Note 1).

    x.2 Test Procedure: A Bayesian Reliability Testing approach will be used for reliability acceptance as follows:

    x.2.1 The data from the reliability qualification test will be used to establish the first prior for the reliability acceptance tests. So long as all the reliability acceptance tests are successful, the test data for all completed tests will be combined with the reliability qualification test data to establish the prior for the next test. Each prior will be tested for adequacy by formula 4.5 of RADC-TR-_____ (SEE NOTE 2). Until an adequate

prior is obtained, the reliability acceptance tests shall be performed using test plan _____ of MIL-STD-781C. (See Note 3). When the prior is adequate the following procedure will be used:

x.2.1.1.   Formulas 4.2 - 4.4 of RADC-TR-_____ (SEE NOTE 2) will be used to determine the prior parameters (λ) and (γ). The test parameter (γ^*) will be computed by the formula $\gamma^* = \gamma/\theta_0$.

x.2.1.2.   The values of (γ^*) and (λ) will be used with table _____ of RADC-78-241 (See Note 4) to determine the test time (T^*) in multiples of θ_0 and the allowable number of failures (r^*). In using the table, the following procedures will be followed:

x.2.1.2.1. If the value of (γ^*), (λ) or both fall outside the limits of the table, formulas 5.7 and 5.8 of RADC-TR-_____ (SEE NOTE 3). will be used to compute values within the table limits, which shall be used in lieu of the original values.

x.2.1.2.2. A table entry of (00.0) for (r^*, t^*) shall not result in acceptance without testing. Instead the sample will be tested for _____ total hours with _____ failures allowed. (See Note 5).

x.2.1.2.2. A table entry of (*****) reflects an unfavorable prior and should not be accessed under these requirements. Its effect would be the same as a failed test.

x.2.2.   Reliability acceptance of production units shall cease whenever a reliability acceptance test reaches a reject decision.   In such event:

x.2.2.1.   The   contractor   shall   propose   corrective   action   for deficiencies in the equipment in accordance with para 5.9 of MIL-STD-781C. Following approval and implementation of this action, the reliability qualification test will be rerun. On its successful completion, reliability

acceptance tests will be resumed as scheduled, but only data from the latest reliability qualification test will be used to establish the first prior, and only data from successful reliability acceptance tests following the latest reliability qualification test will be used to update the prior. (SEE NOTE 6).

NOTES TO USERS

1. Specify how many samples and how often the test will be run, (e.g. -five samples will be chosen randomly from each month's production and the test begun on the first day of the next month. The test will run continuously on all units, except for necessary repairs, until an accept/reject decision is reached).

2. Insert the TR number assigned to this report (shown on the front cover).

3. Any plan with the same discrimination ratio (θ_0/θ_1) as the reliability qualification test can be used, including a repeat of the RQT itself. Suggest a plan with more generous risks (i.e., if the RQT used 10% risks, consider using 20% risks) to reduce test time.

4. The appropriate table must be selected to match the desired discrimination ratio (θ_0/θ_1), and the desired types of consumers and producers risks, and the desired value of the risks. If $\theta_0/\theta_1 = 2$, and average risks of 10% are desired reference may be made to Appendix I of this report instead of to RADC-TR-78-241, thus eliminating one reference document.

5. This is the minimum test used where all previous tests were highly favorable. Your choice of test. A suggestion could be: test for θ_1 with one failure allowed.

6. The contract data requirements list must include appropriate items for the test plan. Test procedures, test report, corrective action plan and any other documentation (e.g., failed item analysis) you desire. See para 30, Appendix F of MIL-STD-781C for data item numbers.

APPENDIX IV

The following reports by RADC on Bayesian Reliability Demonstration are available from:

National Technical Information Service
Department of Commerce
5285 Port Royal Road
Springfield VA 22151

When ordering documents from NTIS, their reference number (AD Numbers shown in parenthesis following the RADC-TR Number) should be cited.

1. Hughes Aircraft Company, "Bayesian Reliability Demonstration: Phase I - Data for the Priori Distributions," RADC-TR-69-389 (AD-866166)

2. Feduccia, A. J., "A Bayesian/Classical Approach to Reliability Demonstration," RADC-TR-70-72 (AD-871969)

3. Hughes Aircraft Company, "Bayesian Reliability Demonstration: Phase II - Development of A Priori Distribution," RADC-TR-71-209 (AD-732283)

4. Hughes Aircraft Company, "Bayesian Reliability Demonstration: Phase III - Development of Test Plans," RADC-TR-73-139 (AD-765172/2)

5. Syracuse University, "Reliability Acceptance Sampling Plans Based Upon Prior Distribution," RADC-TR-76-294 Vols I-V (Vol I - AD-A033515, Vol II - AD-A033516, Vol III - AD-A033517, Vol IV - AD-A033576, Vol V - AD-A033518)

6. Syracuse University, "Design of Reliability Test Plans Based Upon Prior Distribution," RADC-TR-78-241 (AD-A064142)

7. SUNY, "Application of Bayesian Techniques to Reliability Demonstration; Estimation and Updating of the Prior Distribution," RADC-TR-79-121 (AD-A070236)

8. SCEEE, "Bayesian Reliability Theory for Repairable Equipment," RADC-TR-80-30 (AD-A083009)